Using a graphic display calculator

CHAPTER OBJECTIVES:

This chapter shows you how to use your graphic display calculator (GDC) to solve the different types of problems that you will meet in your course. You should not work through the whole of the chapter – it is simply here for reference purposes. When you are working on problems in the mathematical chapters, you can refer to this chapter for extra help with your GDC if you need it.

Instructions for the Casio fx-9860GII calculator

Use this list to help you find the topic you need

Table of contents

Before you start

You should be familiar with:

- Important keys on the keyboard: AC/ON SHIFT DEL MENU X,θ,T EXE OPTN EXIT
- The home screen
- Using the Main Menu to enter the mode you want
- Changing window settings in GRAPH mode
- Using zoom tools in GRAPH mode
- Using trace in GRAPH mode

For a reminder of how to perform the basic operations have a look at your GDC manual.

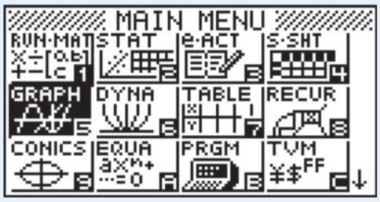
1 Functions

1.1 Graphing linear functions

Example 1

Draw the graph of the function $y = 2x + 1$.

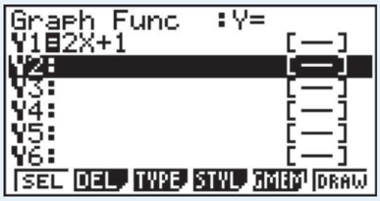
Press MENU. You will see the dialog box as shown on the right.
Choose 5: GRAPH and press EXE.



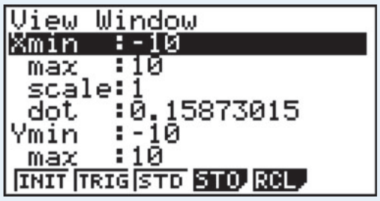
The default graph type is Function, so the form Y= is displayed.

Type $2x + 1$ and press EXE.

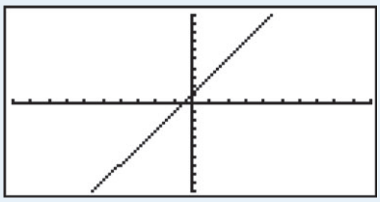
Use X,θ,T to enter x.



Press SHIFT F3 V-Window and choose F3 STD to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.
Press EXE and F6 DRAW.



The graph of $y = 2x + 1$ is now displayed on the screen.



Finding information about the graph

The GDC can give you a lot of information about the graph of a function, such as the coordinates of points of interest and the gradient (slope).

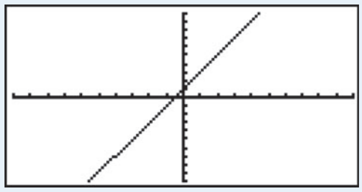
1.2 Finding the gradient (slope) of a line

The correct mathematical notation for gradient (slope) is $\frac{dy}{dx}$. You will find out more about this in the chapter on differential calculus. Here we just need to know this is the notation that will give us the gradient (slope) of the line.

Example 2

Find the gradient of $y = 2x + 1$.

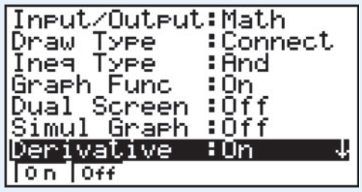
First draw the graph of $y = 2x + 1$ as in Example 1.



Press SHIFT (SET UP).

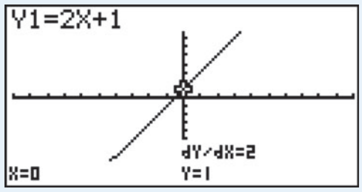
Set Derivative to On.

Press EXE and F6 DRAW to return to the graph.



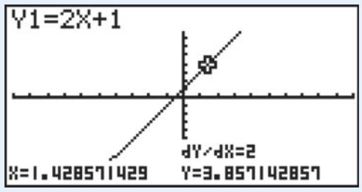
Press SHIFT F1 Trace.

The calculator displays the coordinates of the point and the gradient.



Move the point along the line using the ◀ and ▶ keys.

The gradient (slope) is shown by dy/dx and is 2 at every point along the line.



1.3 Solving simultaneous equations graphically

To solve simultaneous equations graphically you draw the straight lines and then find their point of intersection. The coordinates of the point of intersection give you the solutions x and y.

Solving simultaneous equations using a non-graphical method is covered in section 1.5.

Note: The calculator will only draw the graphs of functions that are expressed explicitly. By that we mean functions that begin with '$y =$' and have a function that involves only x to the right of the equals sign. If the equations are written in a different form, you will need to rearrange them before using your calculator to solve them.

Example 3

Solve the simultaneous equations $2x + y = 10$ and $x - y = 2$ graphically with your GDC.

First rearrange both equations in the form $y =$

$2x - y = 10$	$x - y = 2$
$y = 10 - 2x$	$-y = 2 - x$
	$y = x - 2$

▶ Continued on next page

To draw graphs $y = 10 - 2x$ and $y = x - 2$:

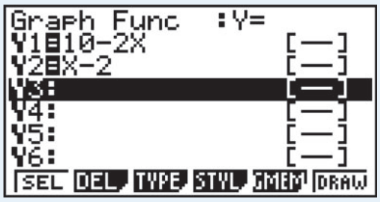
Press MENU and choose 5: GRAPH and press EXE.

The default graph type is Function, so the form Y= is displayed.

Type $10 - 2x$ and press EXE and $x - 2$ and press EXE.

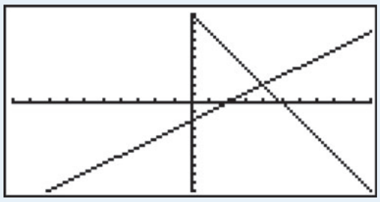
Press SHIFT F3 V-Window and choose F3 STD to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.

Press EXE and F6 DRAW

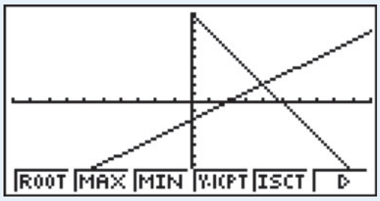


If the calculator displays a graph, press EXIT to return to this screen.

The calculator displays both straight line graphs Y1 = $10 - 2x$ and Y2 = $x - 2$.

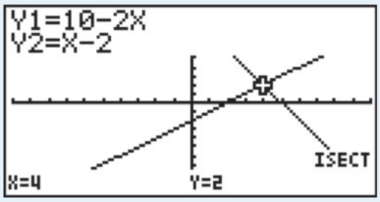


Press F5 G-Solv and F5 ISCT.



The calculator displays the intersection of the two straight lines at the point(4, 2).

The solutions are $x = 4$, $y = 2$.



Simultaneous and quadratic equations

1.4 Solving simultaneous linear equations in two unknowns

When solving simultaneous equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The calculator will do all the working for you.

Example 4

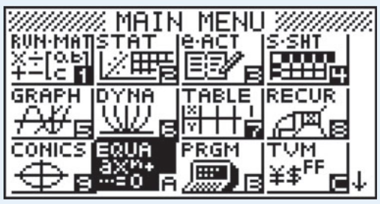
Solve the equations:

$2x + y = 10$

$x - y = 2$

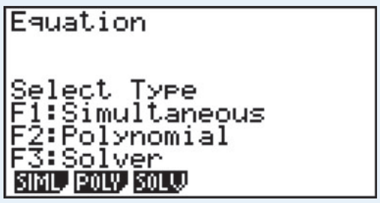
Press MENU. You will see the dialog box as shown on the right.

Choose A: EQUA and press EXE.



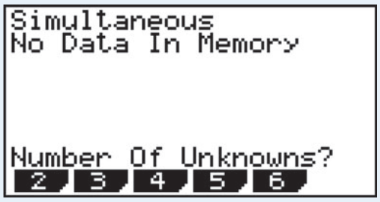
From the menu, choose Simultaneous and press F1.

If there are previous equations in the memory, press EXIT until you return to this menu.



Continued on next page

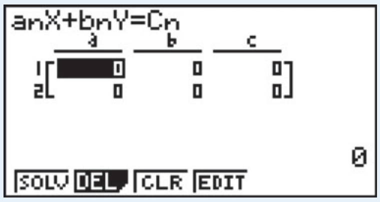
Choose equations in two unknowns and press F1.
Note: This is how you will use the linear equation solver in your examinations. In your project, you might want to solve a more complicated system with more equations and more variables.



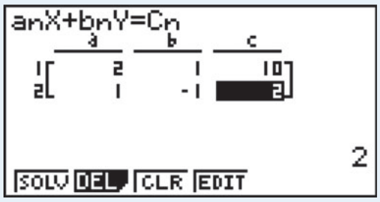
You will see the template on the right.

Type the coefficients from two equations into the template, pressing EXE after each number.

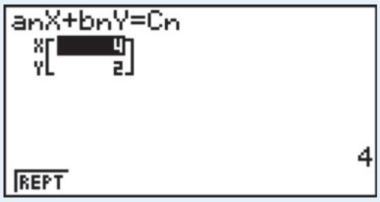
The equations must be in the correct order.



Press F1 and the calculator will solve the equations, giving the solutions as X and Y.

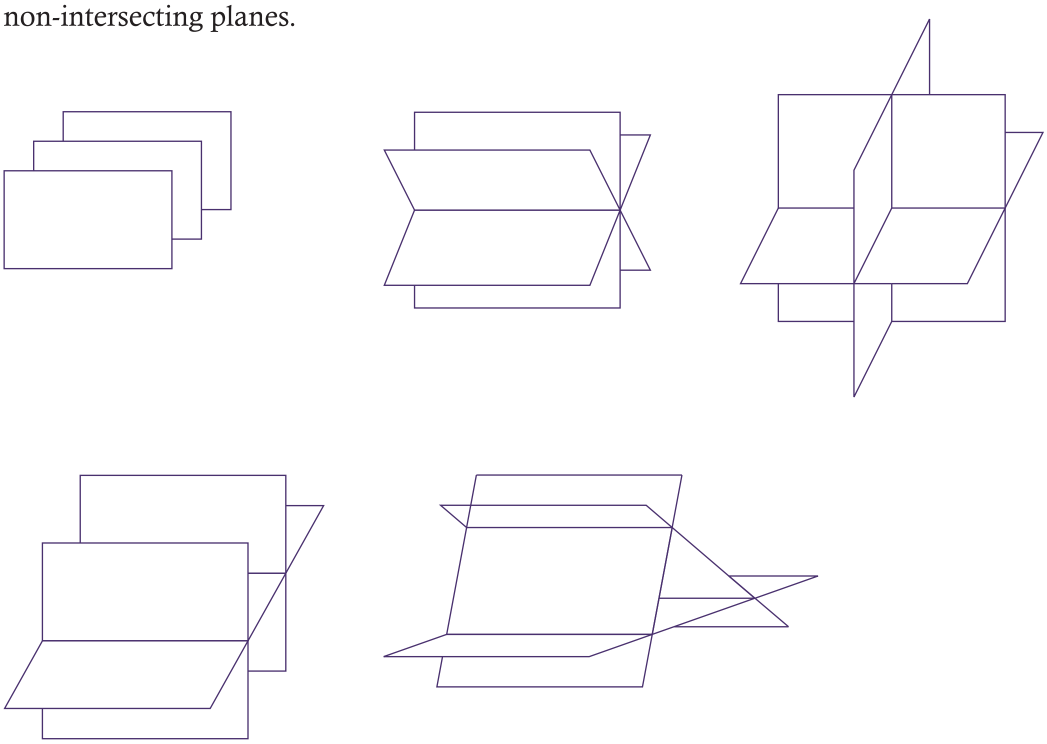


The solutions are $x = 4$, $y = 2$.



1.5 Solving simultaneous linear equations in three unknowns

When solving simultaneous equations in three unknowns there might be a unique solution, infinitely many solutions or no solutions at all. Geometrically, if the equations represent planes in three-dimensions, then their solutions would be intersection at a point, intersection on a line (or plane) or non-intersecting planes.



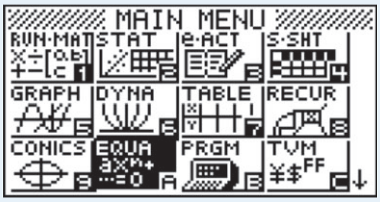
Example 5

Solve the equations

$$2x - 3y + 4z = 1$$
$$x - y - z = -1$$
$$-x + 2y - z = 2$$

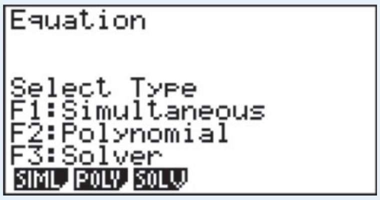
Press MENU. You will see the dialog box as shown on the right.

Choose A: EQUA and press EXE.

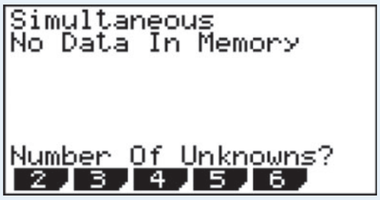


From the menu, choose Simultaneous and press F1.

If there are previous equations in the memory, press EXIT until you return to this menu.



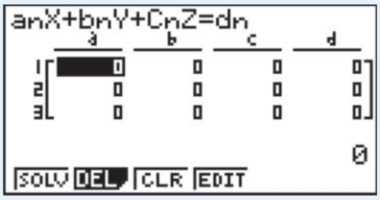
Choose equations in three unknowns and press F2.



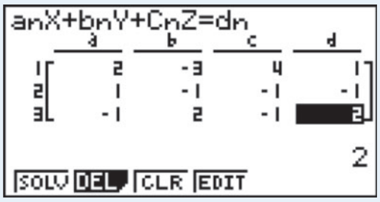
You will see the template on the right.

Type the coefficients from the three equations into the template, pressing EXE after each number.

The equations must be in the correct order.

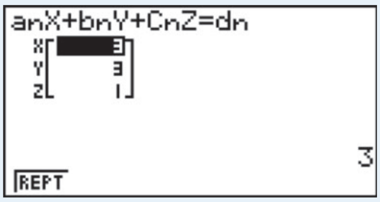


Press F1 and the calculator will solve the equations, giving the solutions as X, Y and Z.



The solutions are $x = 3$, $y = 3$ and $z = 1$

In this example, the solutions represent a point.

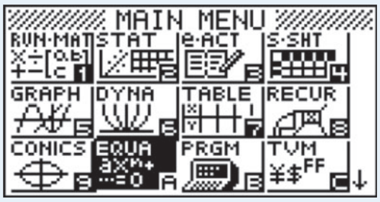


Example 6

Solve the equations

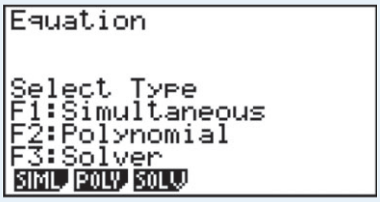
$$2x + 4y + 2z = 8$$
$$x + 2y + z = 4$$
$$3x - y + z = -9$$

Press MENU. You will see the dialog box as shown on the right.
Choose A: EQUA and press EXE.

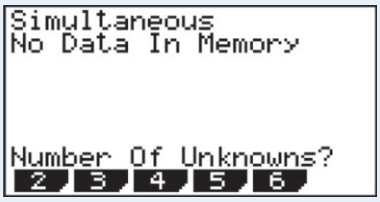


From the menu, choose Simultaneous and press F1.

If there are previous equations in the memory, press EXIT until you return to this menu.

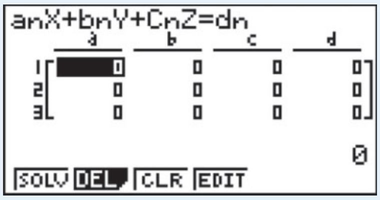


Choose equations in three unknowns and press F2.

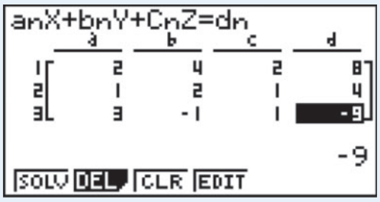


You will see the template on the right.
Type the coefficients from the three equations into the template, pressing EXE after each number.

The equations must be in the correct order.

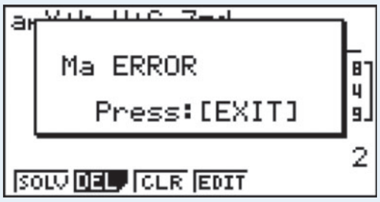


Press F1 and the calculator will solve the equations, giving the solutions as X, Y and Z.

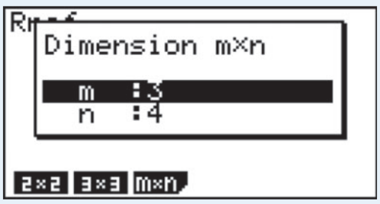


The GDC solves the equations by an inverse matrix method.
In this example there is an error because the matrix is singular, which means that there will be no unique solution.

To find the solution you can use the reduced row echelon form method instead.

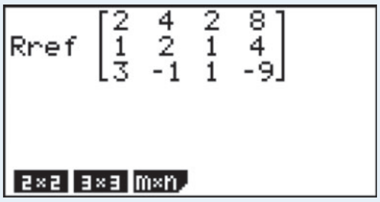


Press MENU 1
Press OPTN | F2 MAT | F6 ▸ | F5 Rref
Press EXIT EXIT | F4 MATH | F1 MAT | F3 m × n
Choose m as 3 and n as 4 and press EXE.



▶ Continued on next page

Type the coefficients from the three equations into the matrix, pressing ▶ after each number.
Press EXE.
The reduced row echelon form gives the solutions to the equations of the line if the final row of the matrix is all zeroes.



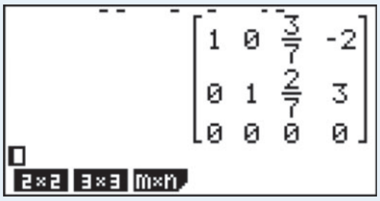
The solutions are equivalent to

$$x+\frac{3}{7}z=-2 \text{ and } y+\frac{2}{7}z=3$$

In this example, the solutions represent a straight line.
The equations of the line can be written

$$\frac{7(x+2)}{-3}=\frac{7(y-3)}{-2}=z$$

$$\frac{x+2}{3}=\frac{y-3}{2}=\frac{z}{-7}$$

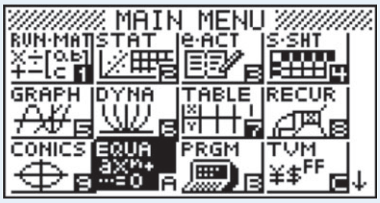


Example 7

Solve the equations

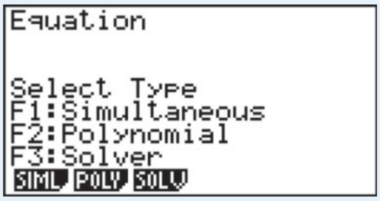
$$x + 2y - 3z = 13$$
$$2x - y + x = 4$$
$$x + 2y - 3z = 7$$

Press MENU. You will see the dialog box as shown on the right.
Choose A: EQUA and press EXE.

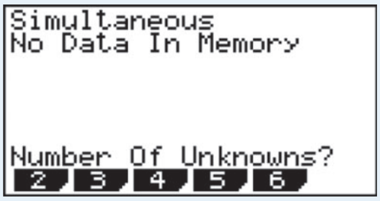


From the menu, choose Simultaneous and press F1.

If there are previous equations in the memory, press EXIT until you return to this menu.

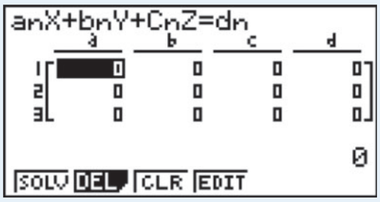


Choose equations in three unknowns and press F2.

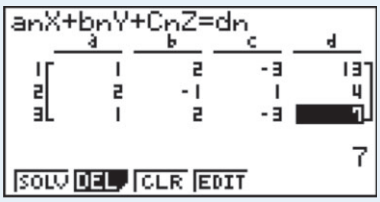


You will see the template on the right.
Type the coefficients from the three equations into the template, pressing EXE after each number.

The equations must be in the correct order.



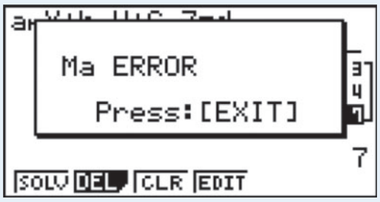
Press F1 and the calculator will solve the equations, giving the solutions as X, Y and Z.



▶ Continued on next page

The GDC solves the equations by an inverse matrix method. In this example there is an error because the matrix is singular, which means that there will be no unique solution.

To find the solution you can use the reduced row echelon form method instead.

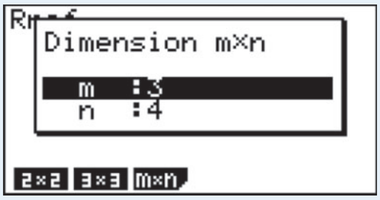


Press MENU 1.

Press OPTN | F2 MAT | F6 ▸ | F5 Rref

Press EXIT EXIT | F4 MATH | F1 MAT | F3 m × n

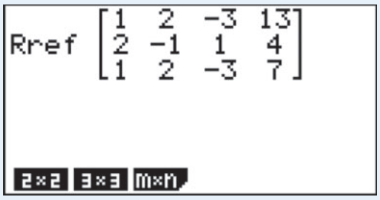
Choose m as 3 and n as 4 and press EXE.



Type the coefficients from the three equations into the matrix, pressing ▸ after each number.

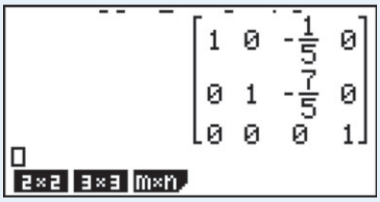
Press EXE.

The reduced row echelon form gives the solutions to the equations of the line if the final row of the matrix is all zeroes.



There are no solutions and the final row is not all zeroes.

In this example the equations are inconsistent.



Quadratic functions

1.6 Drawing a quadratic graph

Example 8

Draw the graph of $y = x^2 - 2x + 3$ and display it using suitable axes.

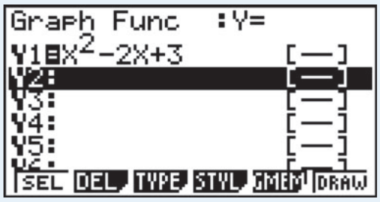
Press MENU and choose 5: GRAPH and press EXE.

The default graph type is Function, so the form Y= is displayed.

Type $y = x^2 - 2x + 3$ and press EXE.

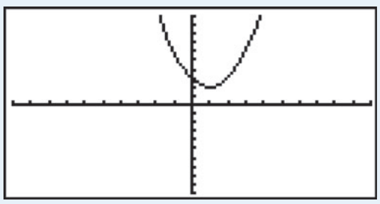
Press SHIFT F3 V-Window and choose F3 STD to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.

Press EXE and F6 DRAW.



If the calculator displays a graph, press EXIT to return to this screen.

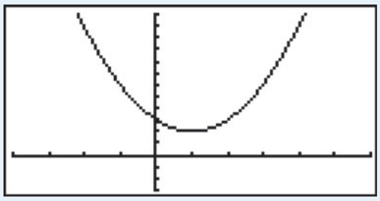
The calculator displays the curve with the default axes.



▶ Continued on next page

Adjust the window to make the exponential curve fit the screen better.

For help with changing axes, see your GDC manual.



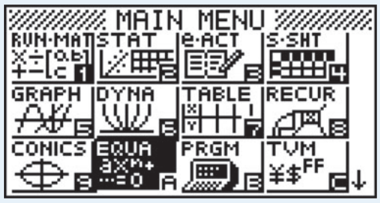
1.7 Solving quadratic equations

When solving quadratic equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The GDC will do all the working for you.

Example 9

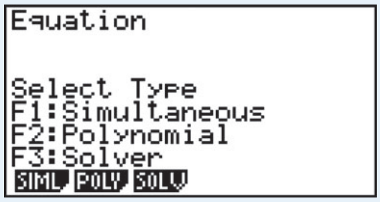
Solve $3x^2 - 4x - 2 = 0$

Press MENU. You will see the dialog box as shown on the right. Choose A: EQUA and press EXE.

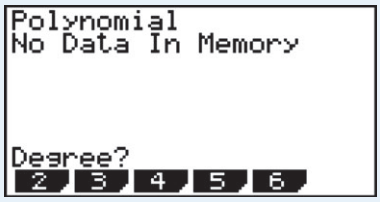


From the menu, choose Polynomial and press F2.

If there are previous equations in the memory, press EXIT until you return to this menu.

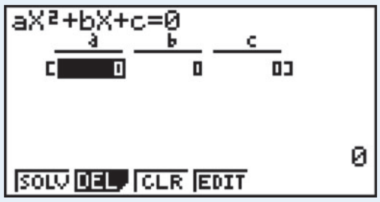


You will see the dialog box as shown on the right. Choose equations of Degree2 and press F1.

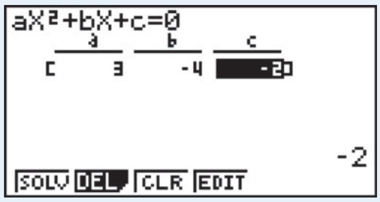


Another dialog box opens for you to enter the equation.

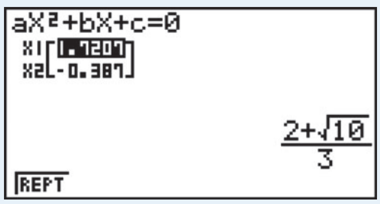
The general form of the quadratic equation is $aX^2 + bX + c = 0$ so we enter the coefficients in a, b and c.



Here $a = 3$, $b = -4$ and $c = -2$. Be sure to use the (–) key to enter the negative values. Press EXE after each value. Press F1 and the calculator will find the roots of the equation.



The solutions are $x = -0.387$ or $x = 1.72$ (3 sf).

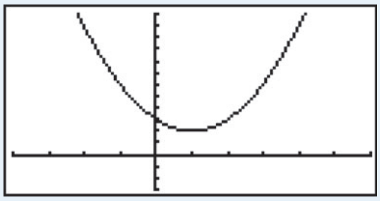


1.8 Finding a local minimum or maximum point

Example 10

Find the minimum point on the graph of $y = x^2 - 2x + 3$.

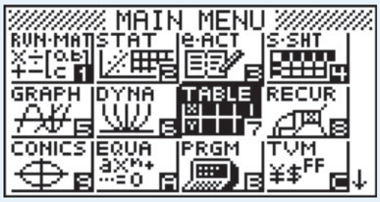
Draw the graph of $y = x^2 - 2x + 3$ (See Example 8).



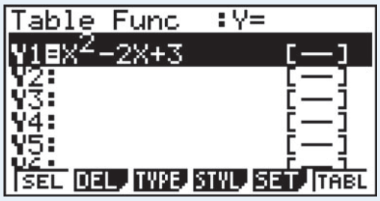
Method 1 - using a table

You can look at a table of the values of the function.

Press MENU and choose 7: TABLE and press EXE.



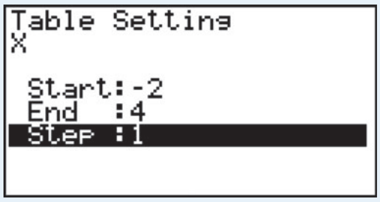
Press F5 SET.



Choose a start and end point for the table and a step of 1.

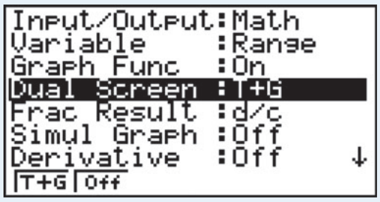
Use the x-values from the graph you drew.

Press EXIT.

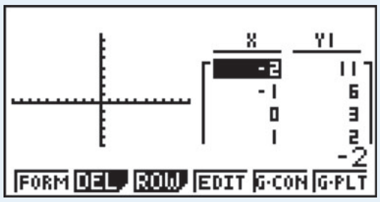


Press SHIFT SET UP.

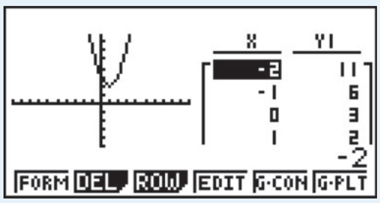
Scroll down to Dual Screen and press F1 T + G.



Press EXE F6 to view the table.

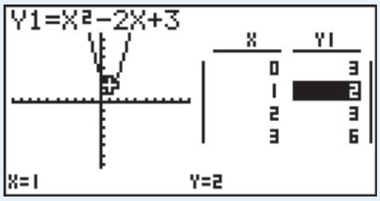


Press F5 G·CON to display the graph alongside the table.

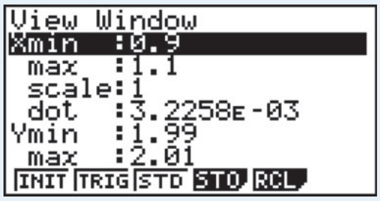


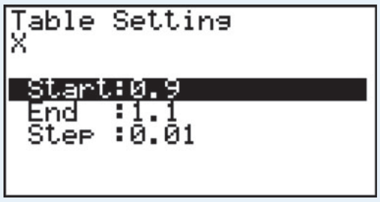
▶ Continued on next page

Press OPTN F2 GLINK.
When you scroll through the table the cross moves along the curve.
The minimum value shown in the table is 2 when $x = 1$.



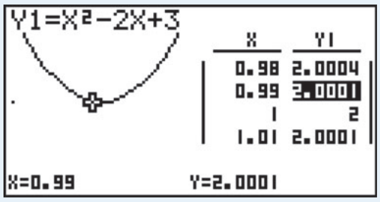
Look more closely at the values of the function around $x = 1$.
Press SHIFT F3 V-Window.
Choose values $-0.9 \le x \le 1.1$ and $1.99 \le y \le 2.01$ and press EXE.
Change the settings in the table: Press F1 F5 SET.
Set Start to 0.9
End to 1.1
Step to 0.01
Press EXE F6 to view the graph and the table.



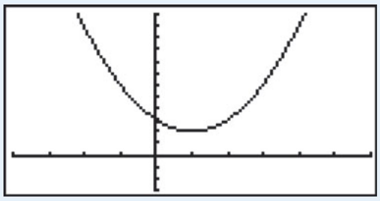


Press F5 G·CON to display the graph alongside the table.
Press OPTN F2 GLINK and scroll through the table.

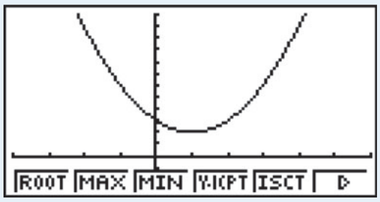
The table shows that the function has larger values at points around (1, 2).
We can conclude that this is a local minimum on the curve.



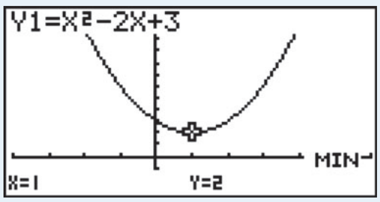
Method 2 - Using the minimum function



Press F5 G-Solv F3 MIN.



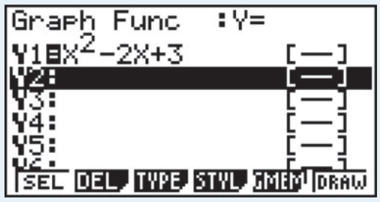
The calculator displays the minimum point on the curve at (1, 2).



Example 11

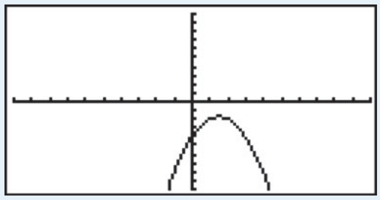
Find the maximum point on the graph of $y = -x^2 + 3x - 4$.

Press MENU and choose 5: GRAPH and press EXE.
The default graph type is Function, so the form Y = is displayed.
Type $-x^2 + 3x - 4$ and press EXE.
Press SHIFT F3 V-Window and choose F3 STD to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.
Press EXE and F6 DRAW.



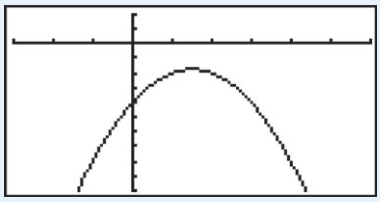
If the calculator displays a graph, press EXIT to return to this screen.

The calculator displays the curve with the default axes.



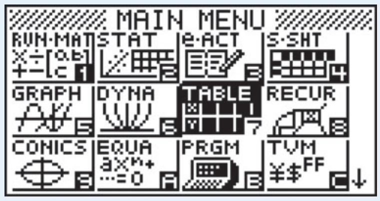
Adjust the window to make the quadratic curve fit the screen better.

For help with changing axes, see your GDC manual.

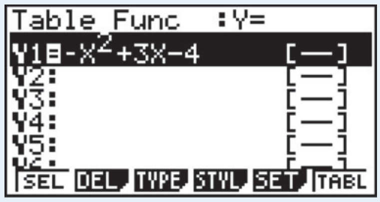


Method 1 - using a table

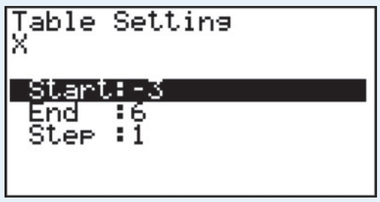
You can look at a table of the values of the function.
Press MENU and choose 7: TABLE and press EXE.



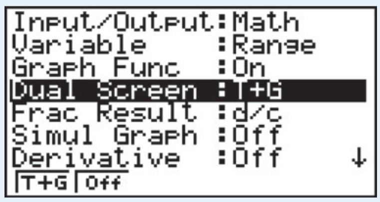
Press F5 SET.



Choose a start and end point for the table and a step of 1.
Use the x-values from the graph you drew.
Press EXIT.

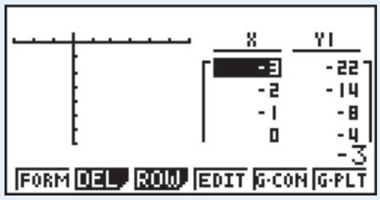


Press SHIFT SET UP.
Scroll down to Dual Screen and press F1 T+G.

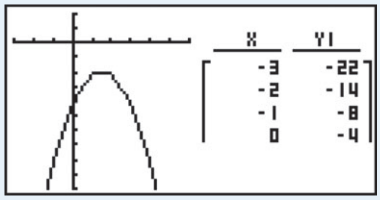


Continued on next page

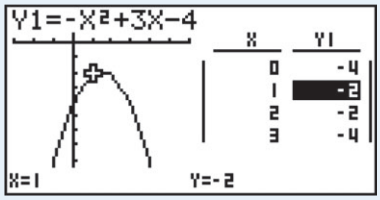
Press EXE F6 to view the table.



Press F5 G·CON to display the graph alongside the table.



Press OPTN F2 GLINK.
When you scroll through the table the cross moves along the curve.
The minimum value shown in the table is –2 when x = 1 and x = 2.



Look more closely at the values of the function around x = 1 and x = 2.

Press SHIFT F3 V-Window.

Choose values $1 \leq x \leq 2$ and $-2.5 \leq y \leq -1.5$ and press EXE.

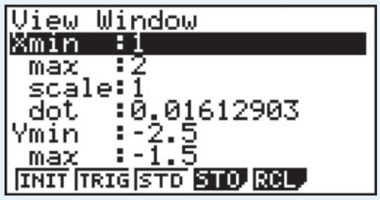
Change the settings in the table: Press F1 F5 SET.

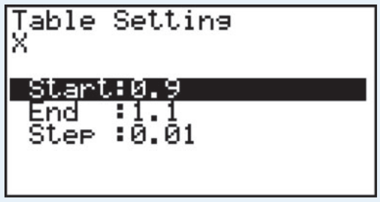
Set Start to 0.9

End to 1.1

Step to 0.01

Press EXE F6 to view the graph and the table.

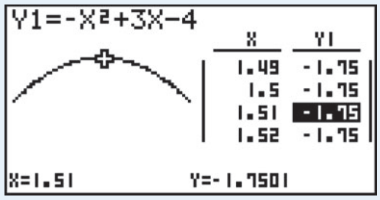




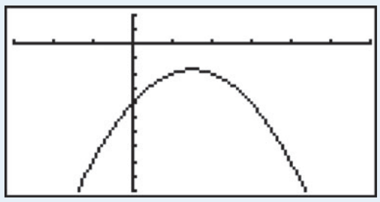
Press F5 G·CON to display the graph alongside the table.

Press OPTN F2 GLINK and scroll through the table.

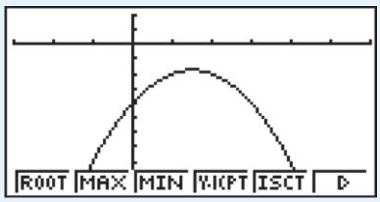
The table shows that the function has larger values at points around (1.5, –1.75). We can conclude that this is a local maximum on the curve.



Method 2 – Using the maximum function

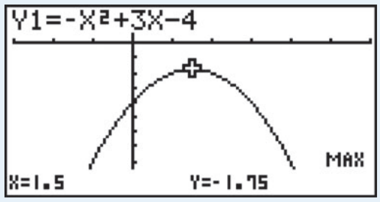


Press F5 G-Solv F3 MIN.



Continued on next page

The calculator displays the maximum point on the curve at (1.5, −1.75).



Complex numbers

1.9 Operations with complex numbers

Example 12

Evaluate the following expressions

i $2(7+i)+\frac{1}{2}(4-2i)$

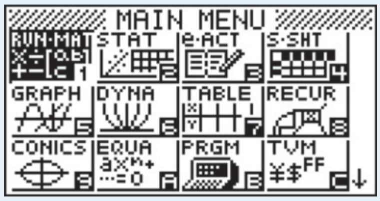
ii $(2+3i)\cdot(3-4i)$

iii $\sqrt{3+4i}$

iv $\frac{1-i}{3+i}$

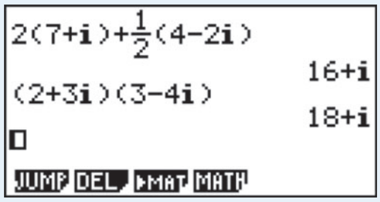
v $(1-i)^3$

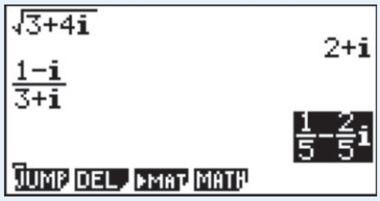
Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.



Complex calculations are entered the same way as you would enter a real expression.
To enter the imaginary number symbol i press SHIFT **i**.
Enter the expressions and then press EXE.

The results are as shown.





1.10 Conjugate, modulus and argument

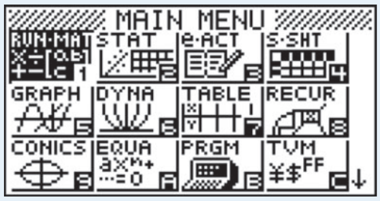
Example 13

Let $z = 1 + \sqrt{3}i$

Find **i** z^* **ii** $|z|$ **iii** arg (z)

Press MENU. You will see the dialog box as shown on the right.

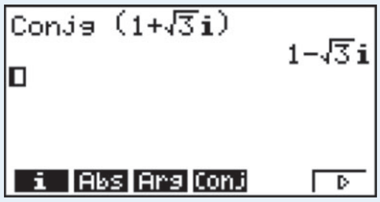
Choose 1: RUN·MAT and press EXE.



i Press OPTN | F3 CPLX | F4 Conj

Enter the complex number in brackets. To enter the imaginary number symbol i press F1 **i**.

Press EXE



ii Press OPTN | F3 CPLX | F2 Abs

Enter the complex number. To enter the imaginary number symbol i press F1 **i**.

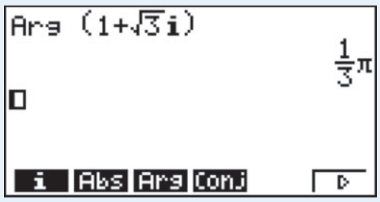
Press EXE

iii Press OPTN | F3 CPLX | F3 Arg

Enter the complex number in brackets.

To enter the imaginary number symbol i press F1 **i**.

Press EXE



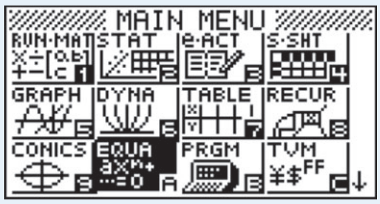
1.11 Solving equations with complex roots

Example 14

Solve the equation $2x^3 - 15x^2 + 44x - 39 = 0$

Press MENU. You will see the dialog box as shown on the right.

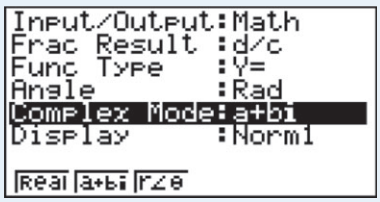
Choose A: EQUA and press EXE.



Press SHIFT SET UP

Select $a + b$**i** as the Complex mode

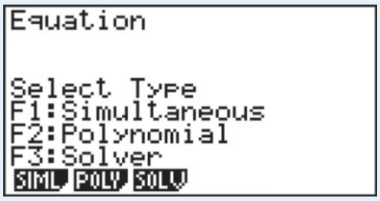
Press EXIT



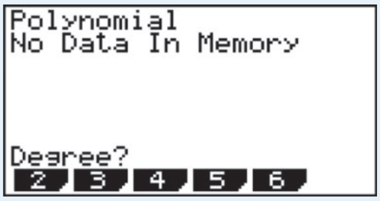
Continued on next page

From the menu, choose Polynomial and press F2.

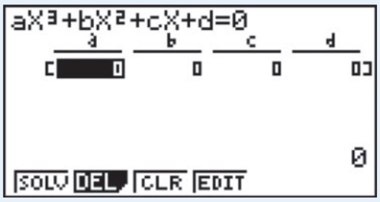
If there are previous equations in the memory, press EXIT until you return to this menu.



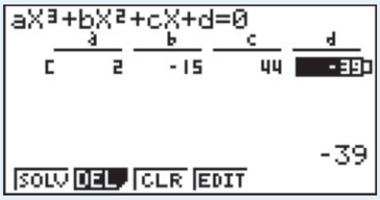
You will see the dialog box as shown on the right.
Choose equations of degree 3 and press F2.



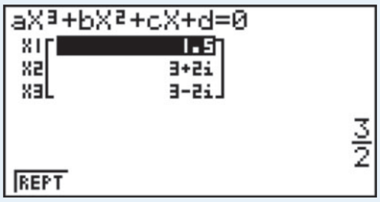
Another dialog box opens for you to enter the equation.
The general form of the quadratic equation is $aX^3 + bX^2 + cX + d = 0$ so we enter the coefficients in a, b, c and d.



Be sure to use the (-) key to enter the negative values.
Press EXE after each value.
Press F1 SOLV and the calculator will find the roots of the equation.



The solutions are,
$x = 1.5$, $x = 3 + 2i$ and $x = 3 - 2i$.



1.12 Polar form

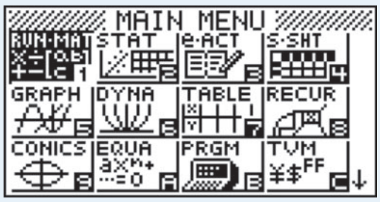
The GDC displays complex numbers in either Cartesian form ($z = x + yi$) or in a modulus, argument form $r\angle\theta$ – see 1.11 for how to find the modulus and argument of a complex number expressed in Cartesian form.

Example 15

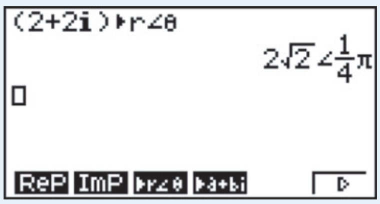
i Change $2 + 2i$ to polar form.

ii Change $3\text{cis}\left(\frac{2\pi}{3}\right)$ to Cartesian form.

Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.
Complex calculations are entered the same way as you would enter a real expression.
To enter the imaginary number symbol i press SHIFT **i**.



i Enter $(2 + 2i)$ and then press OPTN | F3 CPLX | F6 ▶ | F3 ▶ $r\angle\theta$
Press EXE
The result is $2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ or $2\sqrt{2}\,\text{cis}\frac{\pi}{4}$



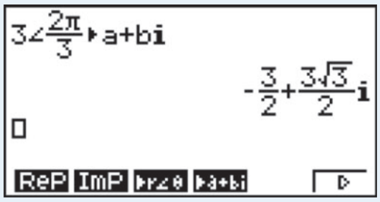
▶ Continued on next page

ii Enter 3 SHIFT $\angle$ a^b/c 2 SHIFT π ▼ 3

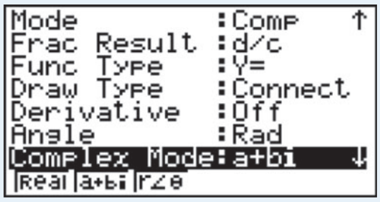
and then press OPTN | F3 CPLX | F6 ▶ | F4 ▶ $a + bi$

Press EXE.

The $\angle$ symbol is above the X,θ,T key



You can also change the mode that the calculator uses to display complex results in settings.
Press SHIFT SET UP
Select Real, Rectangular ($a + bi$) or Modulus Argument ($r \angle\theta$).
For example, in Polar mode, typing $2 + 2i$ EXE would result in the number being displayed in Modulus Argument form without entering F3 ▶ $r \angle\theta$.



Exponential functions

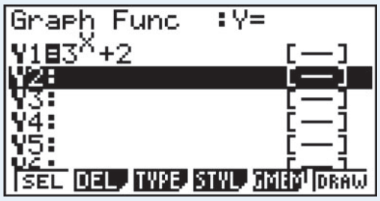
1.13 Drawing an exponential graph

Example 16

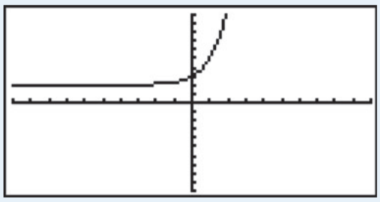
Draw the graph of $y = 3^x + 2$.

Press MENU and choose 5: GRAPH and press EXE.
The default graph type is Function, so the form Y= is displayed.
Type $3^x + 2$ and press EXE.
Press SHIFT F3 V-Window and choose F3 STD to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.
Press EXE and F6 DRAW.

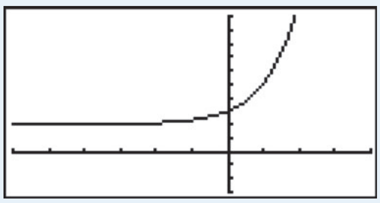
Note: Type 3 ∧ X,θ,T ▶ to enter 3^x.
The ▶ returns you to the baseline from the exponent.



The calculator displays the curve with the default axes.



Adjust the window to make the exponential curve fit the screen better.

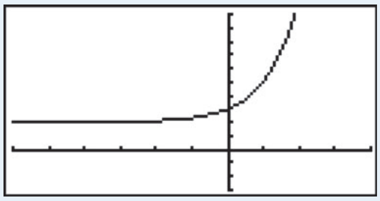


1.14 Finding a horizontal asymptote

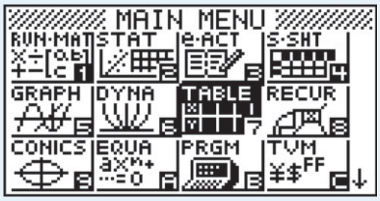
Example 17

Find a horizontal asymptote to the graph of $y = 3^x + 2$.

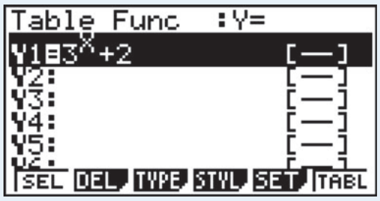
Draw the graph of $y = 3^x + 2$ (See Example 16).



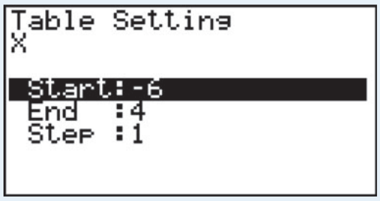
You can look at a table of the values of the function.
Press MENU and choose 7: TABLE and press EXE.



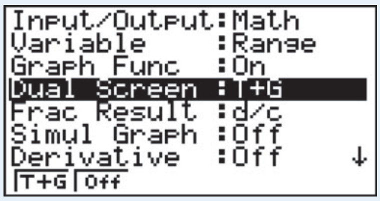
Press F5 SET



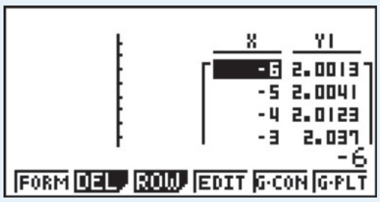
Choose a start and end point for the table and a step of 1.
Use the x-values from the graph you drew.
Press EXIT.



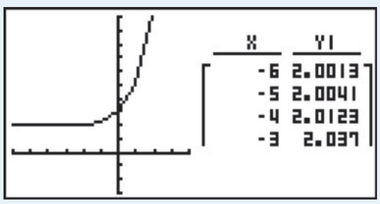
Press SHIFT SET UP.
Scroll down to Dual Screen and press F1 T + G.



Press EXE F6 to view the table.

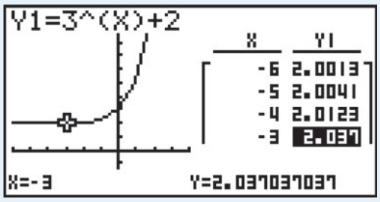


Press F5 G·CON to display the graph alongside the table.

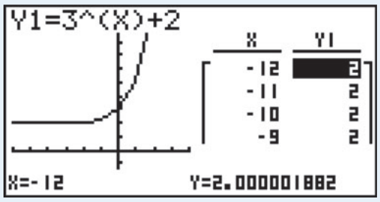


Continued on next page

Press OPTN F2 GLINK.
As the value of x gets smaller, Y_1 gets closer and closer to 2.



Press EXIT and then press F5 SET.
Change the minimum value of x to –12.
EXE F6 F5 G·CON OPTN F2 GLINK.
Eventually the value of Y_1 displayed in the table reaches 2.
You can see, at the bottom of the screen, that the actual value of Y_1 is 2.000001882...
We can say that $Y_1 \to 2$ as $x \to -\infty$.
The line $x = 2$ is a horizontal asymptote to the curve $y = 3^x + 2$.



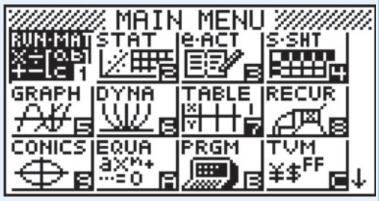
Logarithmic functions

1.15 Evaluating logarithms

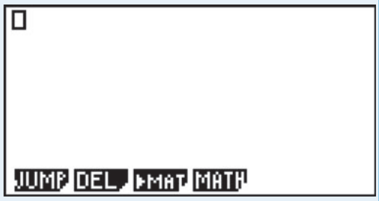
Example 18

Evaluate $\log_{10} 3.95$, $\ln 10.2$ and $\log_5 2$.

Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.

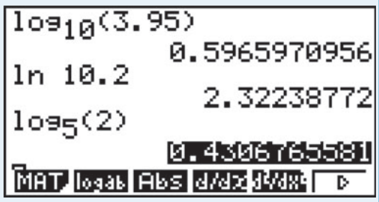


Press EXIT until you see the menu on the right, then press F4 MATH.



Press F2 logab to open the logarithm template.

For natural logarithms it is possible to use the same method, with the base equal to e, but it is quicker to press ln. Similarly for logarithms to base 10 you can press log.
Note that the GDC will evaluate logarithms with any base without having to use the change of base formula.



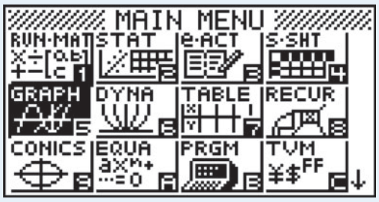
1.16 Finding an inverse function

The inverse of a function can be found by interchanging the x and y values. Geometrically this can be done by reflecting points in the line $y = x$.

Example 19

Show that the inverse of the function $y = 10^x$ is $y = \log_{10}x$ by reflecting $y = 10^x$ in the line $y = x$.

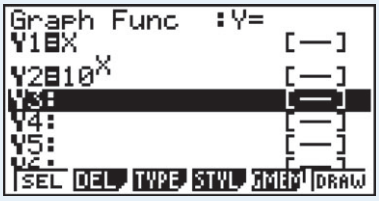
Press MENU. You will see the dialog box as shown on the right.



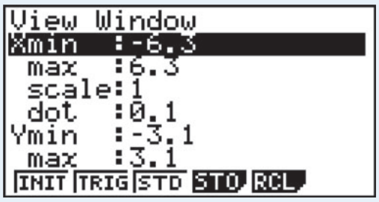
Draw the line $y = x$ so that it can be recognised as the axis of reflection.
Choose 5: GRAPH and press EXE.
Type x and press EXE.
Type 10^x and press EXE.

Use X,θ,T to enter x.

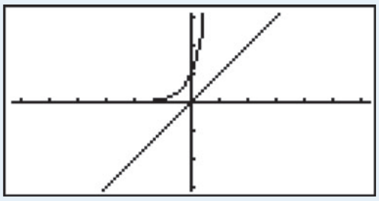
Note: Type 10 1 0 ^ X,θ,T to enter 10^x.



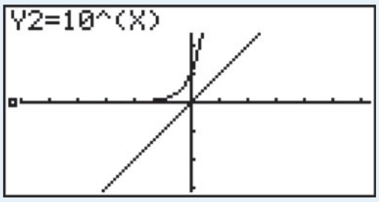
Press SHIFT F3 V-Window and choose F1 INIT to set up square axes which are $-6.3 \le x \le 6.3$ and $-3.1 \le y \le 3.1$.



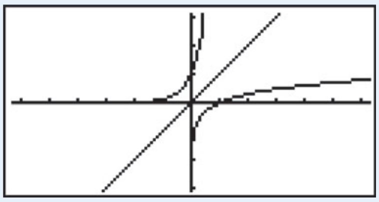
Press EXE and F6 DRAW.
The graphs of $y = x$ and $y = 10^x$ are displayed.



Press SHIFT F4 Sketch | F4 Inv.
Press ▲ to select the curve $Y2 = 10^x$.

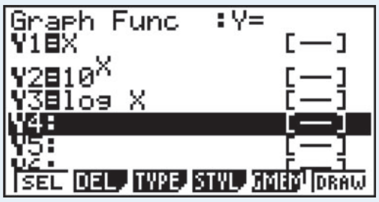


Press EXE.
The calculator will display the inverse of the function $y = 10^x$.

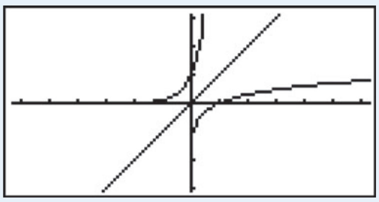


Press EXIT to display the Y= editor.
Type $\log(x)$.

Press log X,θ,T to enter $\log(x)$. log is a shorter way to enter $\log_{10}$.



Press EXE and F6 DRAW to display the graphs of $y = x$, $y = 10^x$ and $y = \log_{10}x$.
The inverse function and the logarithmic function coincide, showing that $y = \log_{10}x$ is the inverse of the function $y = 10^x$.

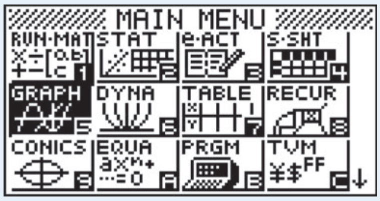


1.17 Drawing a logarithmic graph

Example 20

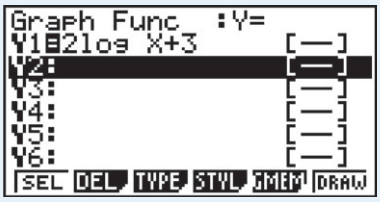
Draw the graph of $y = 2\log_{10}x + 3$.

Press MENU. You will see the dialog box as shown on the right.

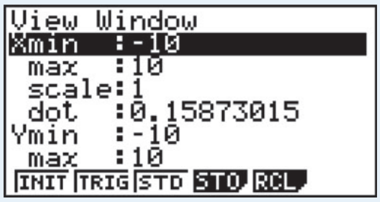


Choose 5: GRAPH and press EXE.
Type 2log(x) + 3 and press EXE.

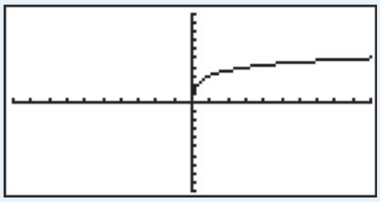
Press 2 log X,θ,T + 3



Press SHIFT F3 V-Window and choose F3 STD for the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

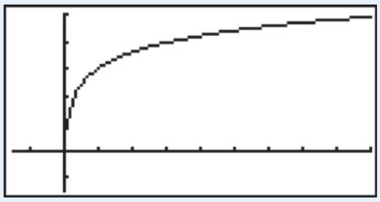


Press EXE and F6 DRAW.
The graphs of $y = 10y = 2\log_{10}x + 3^x$ is displayed with the default axes.



Change the axes to make the logarithmic curve fit the screen better.

For help with changing axes, see your GDC manual.



Trigonometric functions

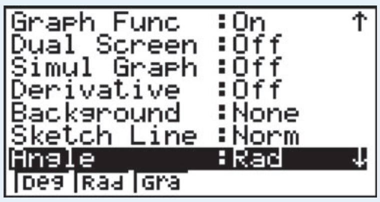
1.18 Degrees and radians

Work in trigonometry will be carried out either in degrees or radians. It is important, therefore, to be able to check which mode the calculator is in and to be able to switch back and forth.

Example 21

Change angle settings from radians to degrees and from degrees to radians.

Press SHIFT SET UP.
Select either Deg or Rad using F1 or F2.
Press EXE.

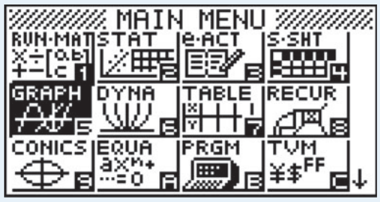


1.19 Drawing trigonometric graphs

Example 22

Draw the graph of $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$.

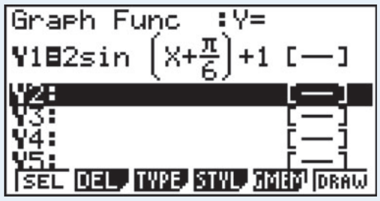
Press MENU. You will see the dialog box as shown on the right.



Choose 5: GRAPH and press EXE.

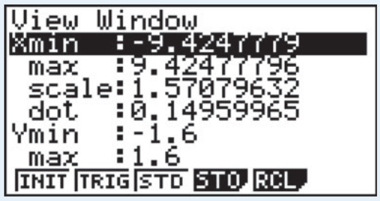
Type $2\sin\left(x + \frac{\pi}{6}\right) + 1$ and press EXE.

Press 2 log X,θ,T + 3



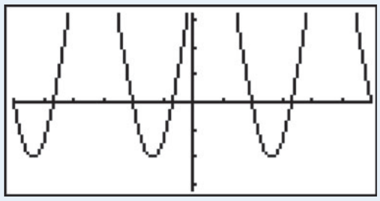
Press SHIFT F3 V-Window and choose F2 TRIG for the default axes which are $-9.42 \leq x \leq 9.42$ and $-1.6 \leq y \leq 1.6$.

$9.42 \approx 3\pi$

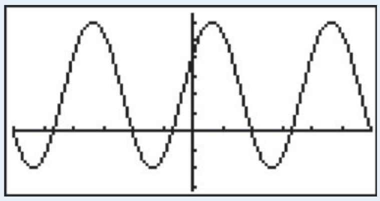


Press EXE and F6 DRAW.

The GDC displays the graph of $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$.



Change the y-axis to make the trigonometric curve fit the screen better.



More complicated functions

1.20 Solving a combined quadratic and exponential equation

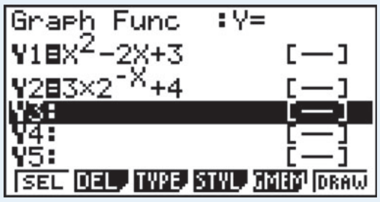
To solve the equation, find the point of intersection between the quadratic function $y_1 = x^2 - 2x + 3$ and the exponential function $y_2 = 3 \times 2^{-x} + 3$.

Example 23

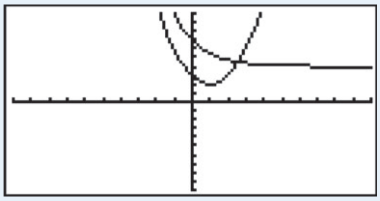
Solve the equation $x^2 - 2x + 3 = 3.2^{-x} + 4$.

Press MENU and choose 5: GRAPH and press EXE.
The default graph type is Function, so the form Y= is displayed.
Type $x^2 - 2x + 3$ in Y_1 and press EXE.
Then type $3 \times 2^{-x} + 4$ in Y_2 and press EXE.
Press SHIFT F3 V-Window and choose F3 STD to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.
Press EXE and F6 DRAW.

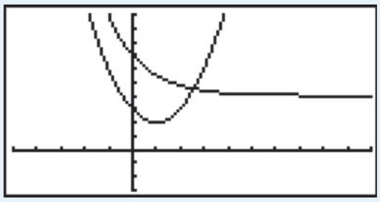
Note: Type 2 ∧ (-) X,θ,T ▶ to enter 2^{-x}. The ▶ returns you to the baseline from the exponent.



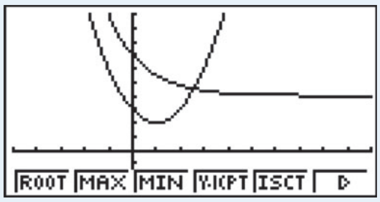
The calculator displays the curves with the default axes.



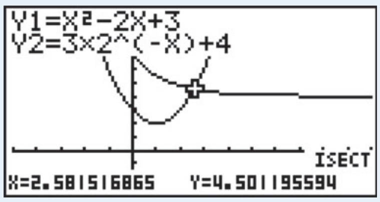
Adjust the window to make the quadratic curve fit the screen better.



Press F5 G-Solv and F5 ISCT.



The calculator displays the intersection of the two straight lines at the point (2.58, 4.50).
The solutions are $x = 2.58$ and $y = 4.50$.



Sequences and series

1.21 Summation of a series

Example 24

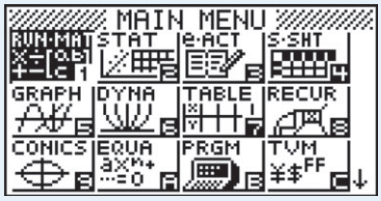
Find the sum of the first 20 terms of the arithmetic sequence 4, 7, 10, 13, …

The kth term of an arithmetic sequence is $u_k = u_1 + (k - 1)d$

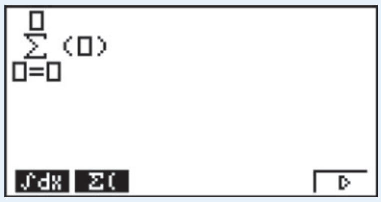
In this example $u_1 = 4$, d = 3 and $n = 20$.

$$S_n = \sum_{k=1}^{20} 4 + (k-1)3$$

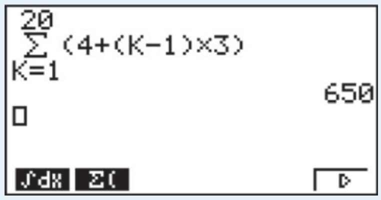
Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.



Press F4 MATH | F6 ▶ | F2 Σ(
The template matches the written Sigma formula.
Enter the variables, values and the function as they are written.
Use the ▶ ◀ ▲ ▼ keys to move around the template.



The sum of the terms of the sequence is 650.



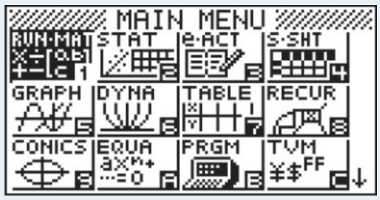
Example 25

Find the sum of the first 12 terms of the geometric sequence $3, -1, \frac{1}{3}, -\frac{1}{9}, \ldots$

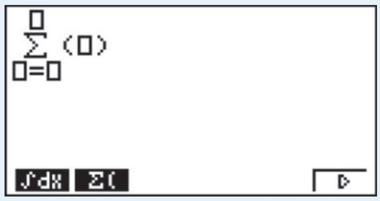
The kth term of a geometric sequence is $u_k = u_1 \cdot r^{k-1}$

In the example $u_1 = 3$, $r = -\frac{1}{3}$ and $n = 12$. $S_n = \sum_{k=1}^{12} 3 \cdot \left(-\frac{1}{3}\right)^{k-1}$

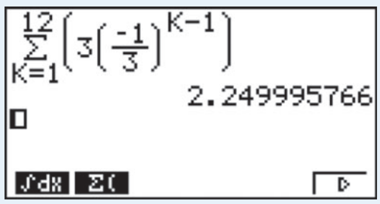
Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.



Press F4 MATH | F6 ▶ | F2 Σ(
The template matches the written Sigma formula.
Enter the variables, values and the function as they are written.
Use the ▶ ◀ ▲ ▼ keys to move around the template.



The sum of the terms of sequence is 2.25, to 3 significant figures.



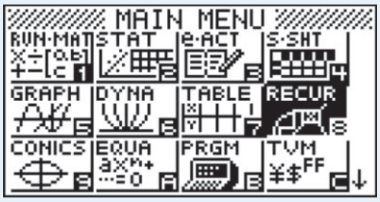
Example 26

How many terms of the series $2 + 1\frac{1}{3} + \frac{8}{9} + \frac{16}{27} + \ldots$ are needed before their sum exceeds 5.5?

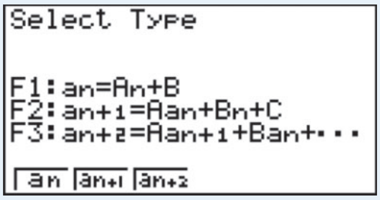
In the example $u_1 = 2$, $r = \frac{2}{3}$ and n is to be found.

$$S_n = \sum_{k=1}^{n} 2 \cdot \left(\frac{2}{3}\right)^{k-1}$$

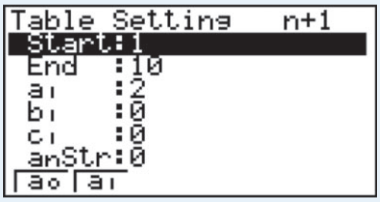
The best way to view a table of the sums of a series is in recursive mode.
Press MENU. You will see the dialog box as shown on the right.
Choose 8: RECUR and press EXE.



Press F3 TYPE and choose F2 $an+1 = Aan + Bn + C$
Press EXIT

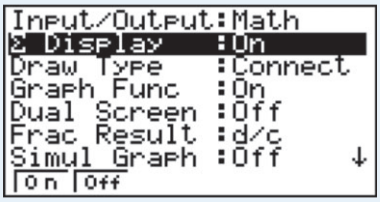


Choose a_{n+1} and press F5 SET
Enter the Start value of the table as 1 and the End value as 10 (this is an estimate of how many terms you will need)
Choose a_1 as the first term and enter 2 as the value of a_1.
Press EXIT

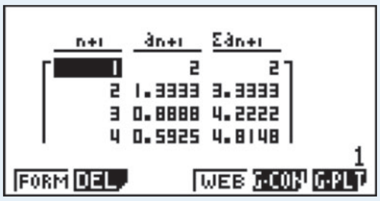


The recursive formula of a geometric series is $u_{n+1} = r \cdot u_n$
Enter this formula using a_{n+1} as $\frac{2}{3} \times a_n$
Type $\frac{2}{3}$ × Press F4 n, a_n·· and choose F2 a_n
Press EXE

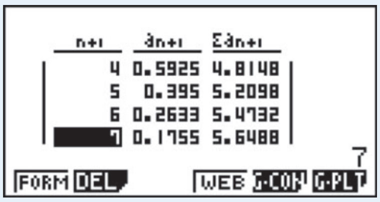
Press SHIFT SET UP
Select Σ Display and press F1 On
This will show the cumulative sums of the terms in the table.
Press EXIT and then F6 TABL



The table shows the first four values of the series and its cumulative sums.
The first row show a_1 and a_1 and the second row shows a_2 and $a_1 + a_2$, etc.



Scrolling down the table shows that when $n = 7$, $S_n > 5.5$ as required.



Modelling

1.22 Using sinusoidal regression

The notation $\sin^2x$, $\cos^2x$, $\tan^2x$, … is a mathematical convention that has little algebraic meaning. To enter these functions on the GDC, you ***should*** enter $(\sin(x))^2$, etc. However, the calculator will conveniently interpret $\sin(x)^2$ as $(\sin(x))^2$.

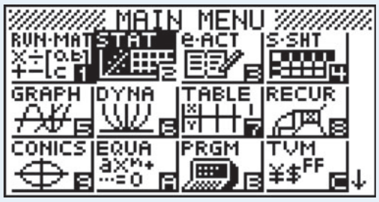
Example 27

It is known that the following data can be modelled using a sine curve.

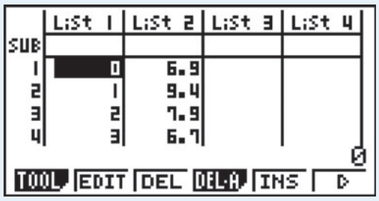
x	0	1	2	3	4	5	6	7
y	6.9	9.4	7.9	6.7	9.2	8.3	6.5	8.9

Use sine regression to find a function to model this data.

Press MENU. You will see the dialog box as shown on the right.
Choose the 2: STAT and press EXE.

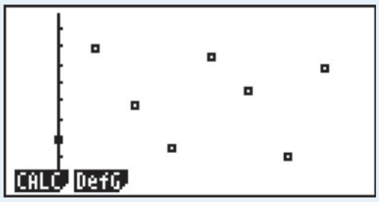


Type the x-values in the first column (List 1) and the y-values in the second column (List 2).
Press EXE after each number to move down to the next cell. Press ▶ to move to the next column.
You can use columns from List 1 to List 26 to enter the lists.

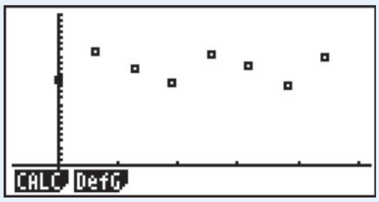


Press F1 GRPH and F6 SET.
Select Graph Type and choose F1 Scatter.
Press EXE.

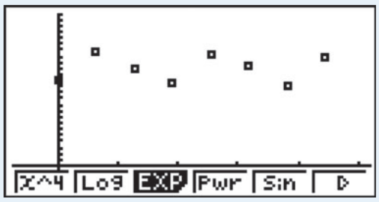
Press F1 GPH1.



The automatic scales do not always give the best display of the box and scatter diagram. You cannot use V-Window to change the default values, but you can zoom in or out.
Press F1 CALC.

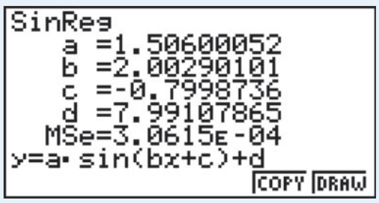


Press F6 ▶ and F5 Sin.

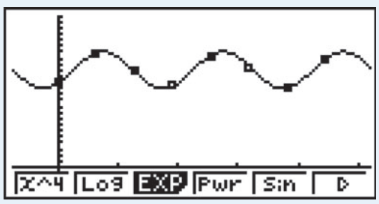


On screen, you will see the result of the sinusoidal regression.
The equation is in the form $y = a\sin(bx + c) + d$ and you will see the values of a, b, c and d displayed separately.

The equation of the sinusoidal regression line is
$y = 1.51\sin(2.00x - 0.80) + 7.99$



Press F6 DRAW to return to the Graphs page.

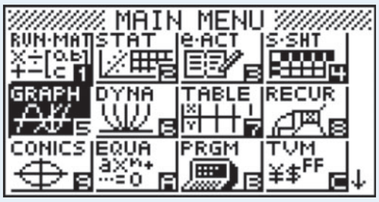


1.23 Drawing a piecewise function

Example 28

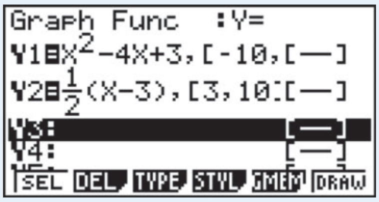
Draw the function $f(x)=\begin{cases} x^2-4x+3, & x<3 \\ \frac{1}{2}(x-3), & x\geq 3 \end{cases}$

Press MENU. You will see the dialog box as shown on the right.
Choose 5: GRAPH and press EXE.

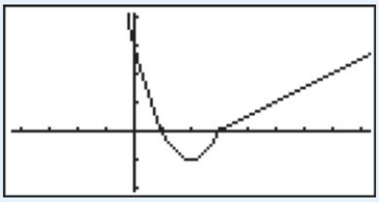


The default graph type is Function, so the form Y= is displayed.
Type x^2-4x+3, [−10, 3] in Y1 and press EXE.
Type $\frac{1}{2}(x-3)$[3, 10] in Y2 and press EXE.

Use square brackets to enter the domains.



Press F6 DRAW
Choose suitable axes to display the curves.
The piecewise function is displayed.



2 2 Differential calculus

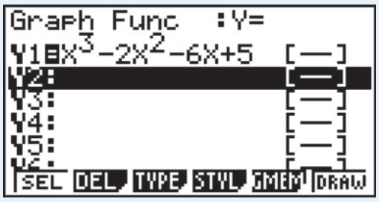
2.1 Finding the gradient at a point

Example 29

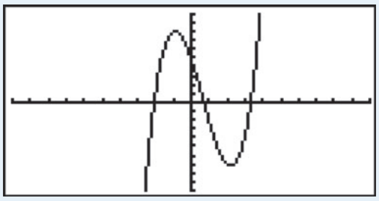
Find the gradient of the cubic function $y = x^3 - 2x^2 - 6x + 5$ at the point where $x = 1.5$.

Press MENU and choose 5: GRAPH and press EXE.
The default graph type is Function, so the form Y= is displayed.
Type $y = x^3 - 2x^2 - 6x + 5$ and press EXE.
Press SHIFT F3 V-Window and choose F3 STD to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.
Press EXE and F6 DRAW.

Note: Type X,θ,T ∧ 3 ▸ to enter x^3.
The ▸ returns you to the baseline from the exponent.



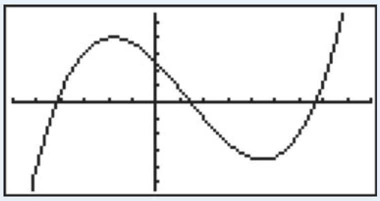
The calculator displays the curve with the default axes.



▶ Continued on next page

Adjust the window to make the cubic curve fit the screen better.

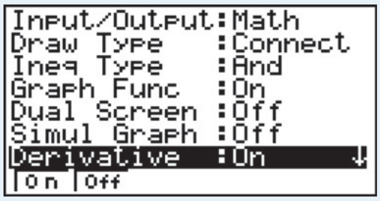
For help with changing axes, see your GDC manual.



Press SHIFT (SET UP).

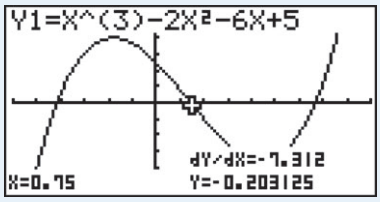
Set Derivative to On.

Press EXE and F6 DRAW to return to the graph.



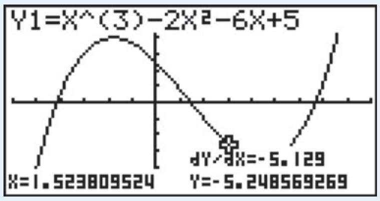
Press SHIFT F1 Trace.

The calculator displays the coordinates of the point and the gradient.

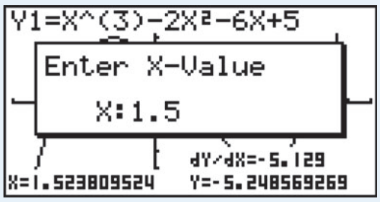


Move the point along the line using the ◂ and ▸ keys.

The gradient (slope) can be found at a point that is close to 1.5.

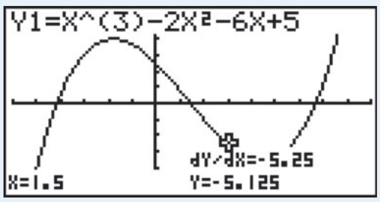


Type 1 . 5 EXE.



The calculator displays the gradient of the curve at the point where $x = 1.5$.

The gradient is –5.25.

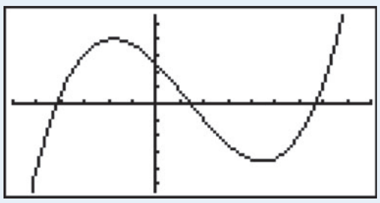


2.2 Drawing a tangent to a curve

Example 30

Draw a tangent to the curve $y = x^3 - 2x^2 - 6x + 5$ where $x = -0.5$.

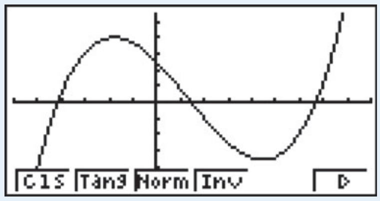
First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 29).



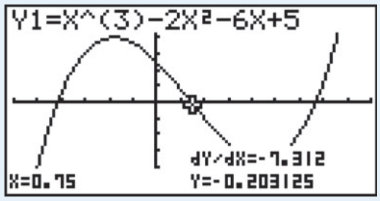
▶ Continued on next page

Press F4 Sketch.

Press F2 Tang.

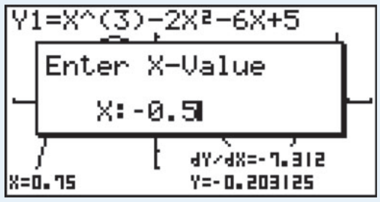


The calculator displays the trace screen.

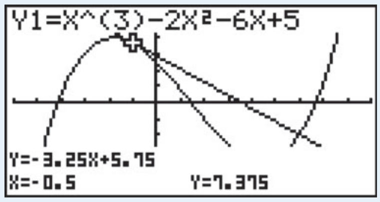


Press (-) 0 . 5 EXE.

Press EXE.



The equation of the tangent is $y = -3.25x + 5.75$.

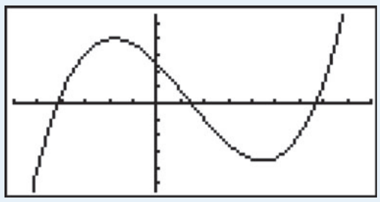


2.3 Finding maximum and minimum points

Example 31

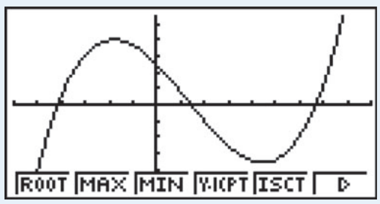
Find the local maximum and local minimum points on the cubic curve.

First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 29).

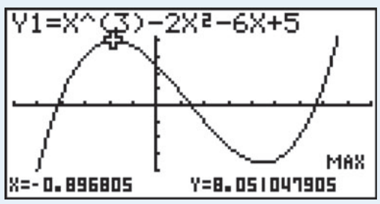


Press SHIFT F5 G-Solv.

Press F2 MAX to find the maximum point.



The calculator displays the local maximum at the point (−0.897, 8.05).

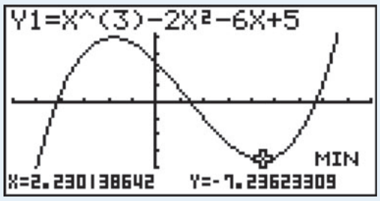


Continued on next page

Press SHIFT F5 G-Solv.

Press F3 MIN to find the minimum point.

The calculator displays the local minimum at the point (2.23, −7.24).



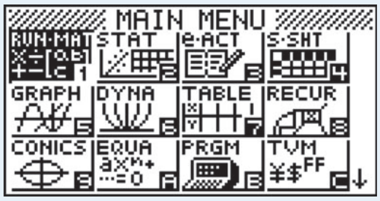
2.4 Finding a numerical derivative

Using the calculator it is possible to find the numerical value of any derivative for any value of x. The calculator will not, however, differentiate a function algebraically. This is equivalent to finding the gradient at a point graphically (see Section 2.1 example 29).

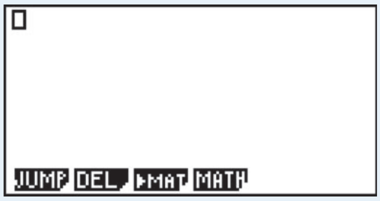
Example 32

If $y=\frac{x+3}{x}$, evaluate $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=2}$

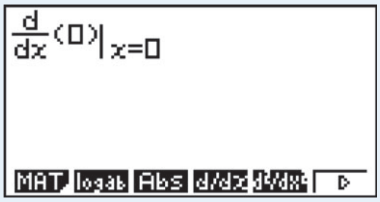
Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.



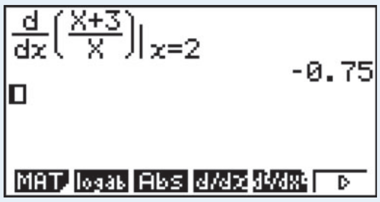
Press EXIT until you see the menu on the right, then press F4 MATH.



Press F4 $\frac{\mathrm{d}}{\mathrm{d}x}$ to open the differentiation template.



The calculator shows that the value of the first derivative of $y=\frac{x+3}{x}$ is −0.75 when $x=2$.



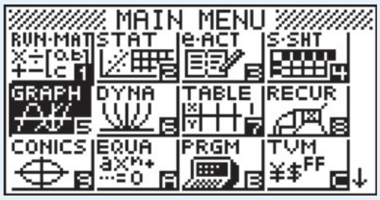
2.5 Graphing a numerical derivative

Although the calculator can only evaluate a numerical derivative at a point, it will graph the gradient function for all values of x.

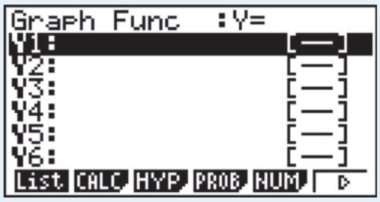
Example 33

If $y = \dfrac{x}{x+3}$, draw the graph of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

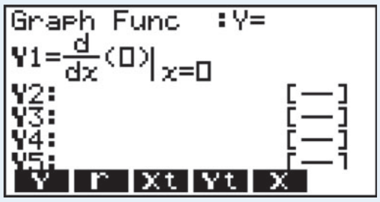
Press MENU. You will see the dialog box as shown on the right.



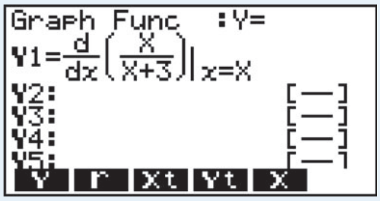
Choose 5: GRAPH and press EXE.
Press OPTN F2 CALC.



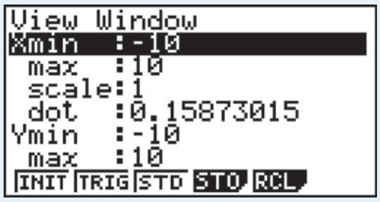
Choose F1 $\dfrac{\mathrm{d}}{\mathrm{d}x}$ to choose the derivative template.



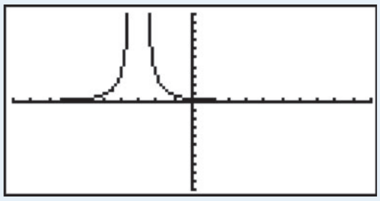
In the template enter x, the function $\dfrac{x}{x+3}$ and the value x.
Press EXE.



Press SHIFT F3 V-Window and choose F3 STD for the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.



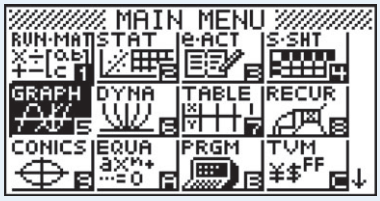
Press EXE and F6 DRAW.
The GDC displays the graph of the numerical derivative function of $y = \dfrac{x}{x+3}$.



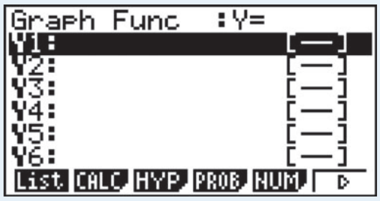
Example 34

Find the values of x on the curve $y = \frac{x^3}{3} + x^2 - 5x + 1$ where the gradient is 3.

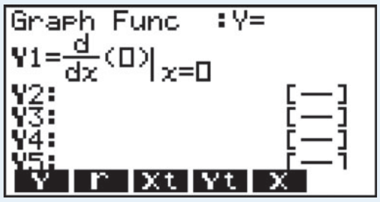
Press MENU. You will see the dialog box as shown on the right.



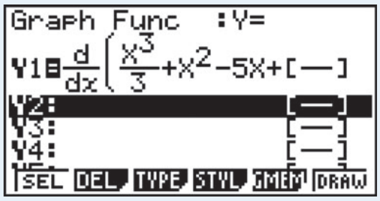
Choose 5: GRAPH and press EXE.
Press OPTN F2 CALC.



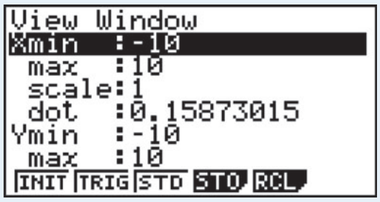
Choose F1 $\frac{\mathrm{d}}{\mathrm{d}x}$ to choose the derivative template.



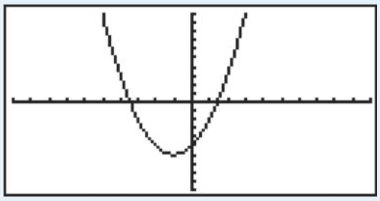
In the template enter x, the function $\frac{x^3}{3} + x^2 - 5x + 1$ and the value x.
Press EXE.



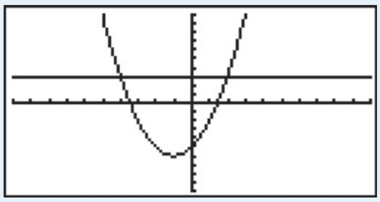
Press SHIFT F3 V-Window and choose STD for the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.



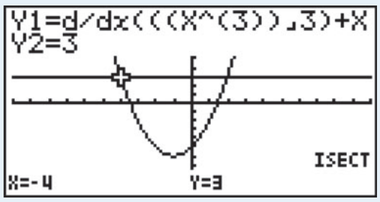
Press EXE and F6 DRAW.
The calculator displays the graph of the numerical derivative function of $y = \frac{x^3}{3} + x^2 - 5x + 1$.



Press EXIT to display the Y= editor.
Enter the function $Y_2 = 3$.
Press F6 DRAW.
The calculator now displays the curve and the line $y = 3$.



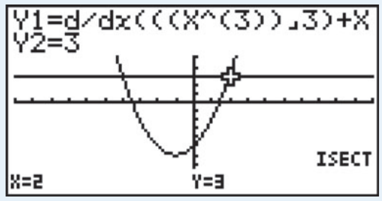
To find the points of intersection between the curve and the line:
Press SHIFT F5 G-Solv.
Press F5 ISCT.
The GDC shows a point of intersection at (4, 3).



Continued on next page

Press ▶ to select the second point.
The GDC shows a point of intersection at (2, 3).

The curve has gradient 3 when $x = -4$ and $x = 2$.



2.6 Using the second derivative

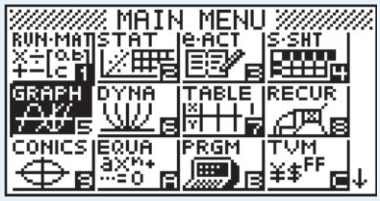
The calculator can find first and second derivatives. The second derivative can be used to determine whether a point is a maximum or minimum point.

Example 35

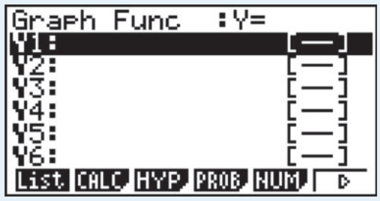
Find the stationary points on the curve $f(x) = x^4 - 4x^3$ and determine their nature.

$f(x) = x^4 - 4x^3$
$f'(x) = 4x^3 - 12x^2$
At stationary points
$f'(x) = 0$
$4x^3 - 12x^2 = 0$
$4x^2(x - 3) = 0$
Therefore $x = 0$ or $x = 3$.

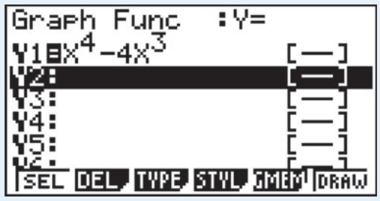
Press MENU. You will see the dialog box as shown on the right.



Choose 5: GRAPH and press EXE.

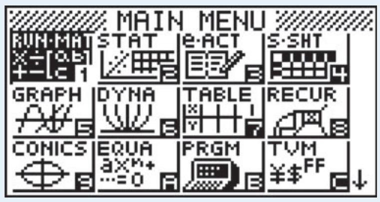


Enter $x^4 - 4x^3$ for Y1.

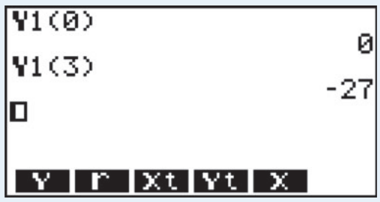


Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.

Use ▲ to enter the exponent.
Press ▶ to return to the base line.

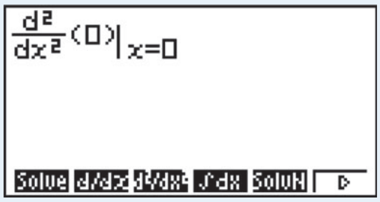


Use the calculator to find the coordinates of the points and to determine their nature.
Evaluate the function when $x = 0$ and $x = 3$.
Press VARS | F4 GRPH | F1 Y then type 1(0)l to enter Y1(0).
Similarly enter Y1(3).
The stationary points are at (0, 0) and (3, −27).

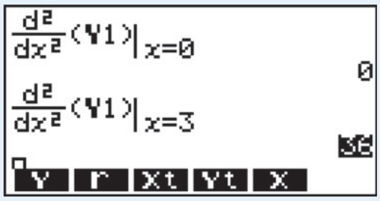


Continued on next page

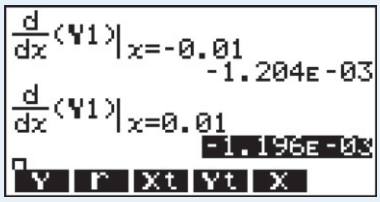
Press OPTN F4 CALC | F3 $\frac{d^2}{dx^2}$ to enter the second derivative template.



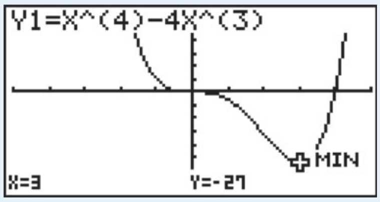
Enter Y1 in the template as the function using the same procedure to enter the Y.
Enter the value of x as 0.
Repeat for the second derivative when $x = 3$.
In this case we are not certain what the nature of the stationary point is at (0, 0) but the point (3, −27) is a minimum because $f''(x) > 0$.



Evaluate $f'(x)$ either side of $x = 0$, in this case using $x = -0.01$ and $x = 0.01$.
To evaluate the derivatives follow the same steps as before, but use F2 $\frac{d}{dx}$.
The gradient is negative either side of the stationary point.
Hence (0, 0) is a negative point of inflection.



The graph on the right illustrates the curve, the minimum at (3, –27) and the point of inflection at (0, 0).



3 Integral calculus

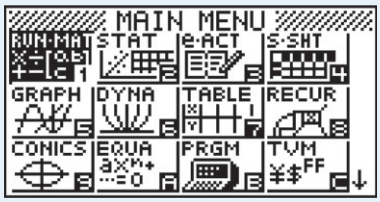
The calculator can find the values of definite integrals either on a calculator page or graphically. The calculator method is quicker, but the graphical method is clearer and shows discontinuities, negative areas and other anomalies that can arise.

3.1 Finding the value of a definite integral

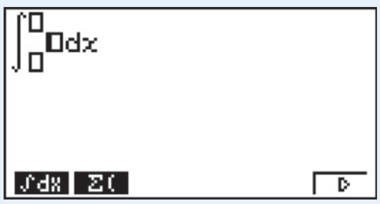
Example 36

Evaluate $\int_2^8 \left(x - \frac{3}{\sqrt{x}}\right) dx$

Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.

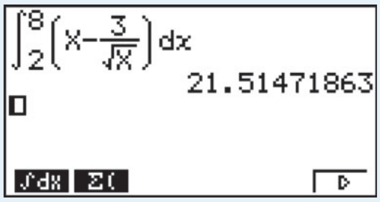


Press OPTN F4 CALC | F6 ▲ | F1 ∫dx to enter the second derivative template.
In this example you will also use templates to enter the rational function and the square root.



Continued on next page

Enter the upper and lower limits, the function and x in the template.
Press EXE.
The value of the integral is 21.5 (to 3 sf).

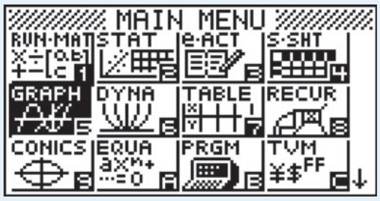


3.2 Finding the area under a curve

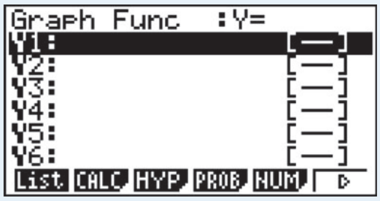
Example 37

$y = 3x^2 - 5$
Find the area bounded by the curve, the x-axis and the lines $x = -1$ and $x = 1$.

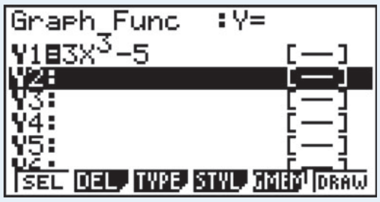
Press MENU. You will see the dialog box as shown on the right.



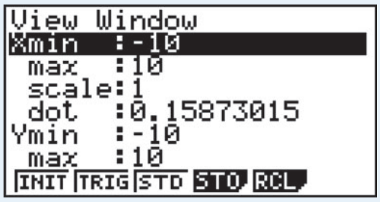
Choose 5: GRAPH and press EXE.
Press OPTN F2 CALC.



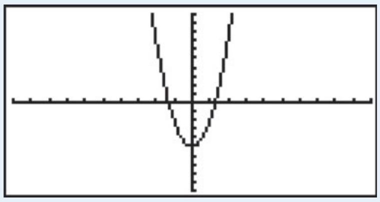
Enter the function $y = 3x^2 - 5$ and press EXE.



Press SHIFT F3 V-Window and choose F3 STD for the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.

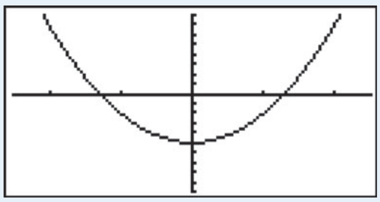


Press EXE and F6 DRAW.
The graph of $y = 3x^2 - 5$ is displayed with the default axes.

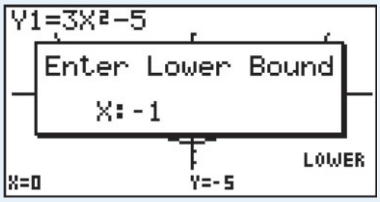
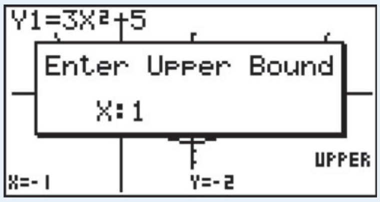
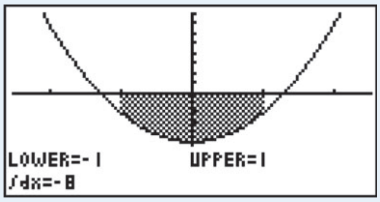


Change the axes to make the curve fit the screen better.

For help with changing axes, see your GDC manual.



Continued on next page

Press F5 G-Solv | F6 ▲ | F3 ∫dx. The calculator prompts you to enter the lower limit for the integral. Type –1 and press EXE. Be sure to use the (–) key.	
The calculator prompts you to enter the upper limit for the integral. Type 1 and press EXE.	
The area found is shaded and the value of the integral (–8) is shown on the screen. The required area is 8. Since the area lies below the x-axis in this case, the integral is negative.	

4 Vectors

There is no scalar product function on the Casio GDC, but you can find the result by multiplying the vectors as lists and then finding the sum of the terms in the list.

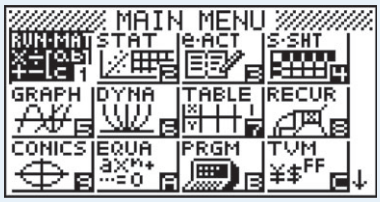
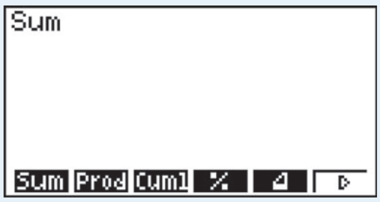
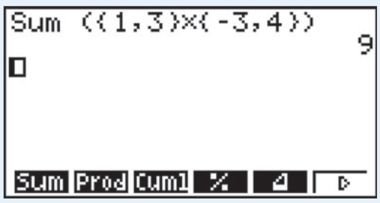
Scalar product

4.1 Calculating a scalar product

Example 38

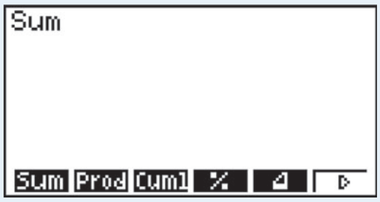
Evaluate the scalar products:

a $\begin{pmatrix}1\\3\end{pmatrix}\cdot\begin{pmatrix}-3\\4\end{pmatrix}$ **b** $\begin{pmatrix}1\\-1\\4\end{pmatrix}\cdot\begin{pmatrix}3\\2\\-1\end{pmatrix}$

Press MENU. You will see the dialog box as shown on the right. Choose 1: RUN·MAT and press EXE.	
Press OPTN F1 LIST | F6 ▶ | F6 ▶ | F1 Sum.	
Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas. Multiply the two vector lists together. Close the bracket and press ENTER. $\begin{pmatrix}1\\3\end{pmatrix}\cdot\begin{pmatrix}-3\\4\end{pmatrix}=9$	

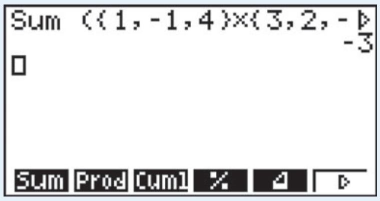
▶ Continued on next page

Press OPTN F1 LIST | F6 ▸ | F6 ▸ | F1 Sum.



Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas.
Multiply the two vector lists together.
Close the bracket and press ENTER.

$$\begin{pmatrix}1\\-1\\4\end{pmatrix}\cdot\begin{pmatrix}3\\2\\-1\end{pmatrix}=-3$$



4.2 Calculating the angle between two vectors

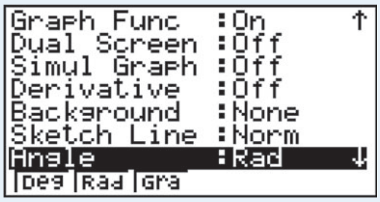
The angle θ between two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, can be calculated using the formula

$$\theta=\arccos\left(\frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|}\right)$$

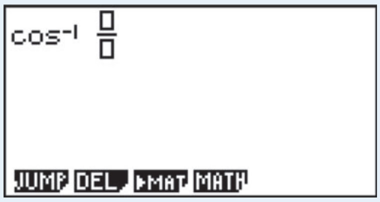
Example 39

Calculate the angle between $2\vec{\mathbf{i}}+3\vec{\mathbf{j}}$ and $3\vec{\mathbf{i}}-\vec{\mathbf{j}}$.

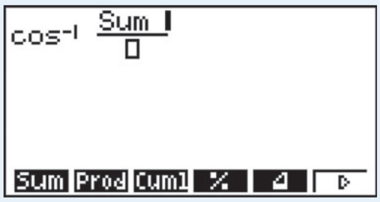
Press SHIFT SET UP.
Select either Deg or Rad using F1 or F2.
Press EXE.



Press SHIFT ($\cos^{-1}$) $a^{b}/_{c}$ to select the fraction template.



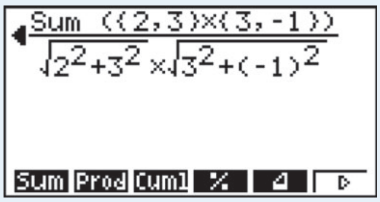
Press OPTN F1 LIST | F6 ▸ | F6 ▸ | F1 Sum.



Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas.
Multiply the two vector lists together.

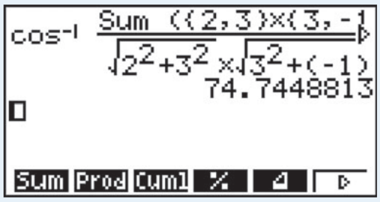
To calculate the magnitudes of the vectors use the formula

$$|a\vec{\mathbf{i}}+b\vec{\mathbf{j}}|=\sqrt{a^2+b^2}$$



▶ Continued on next page

Press EXE.
The angle between $2\vec{i}+3\vec{j}$ and $3\vec{i}-\vec{j}$ is 74.7°.



Vector product

4.3 Calculating a vector product

The Casio fx-9860GII does not have the ability to perform cross products of vectors.

5 Statistics and probability

You can use your GDC to draw charts to represent data and to calculate basic statistics such as mean, median, etc. Before you do this you need to enter the data in a list.

Entering data

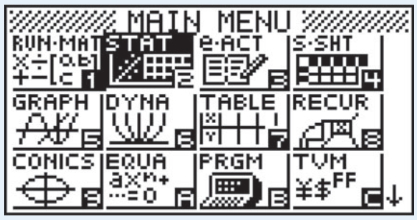
There are two ways of entering data: as a list or as a frequency table.

5.1 Entering lists of data

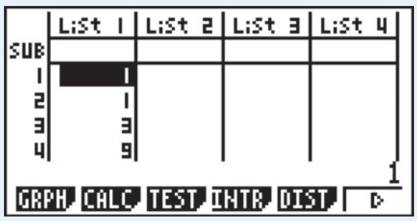
Example 40

Enter the data in the list: 1, 1, 3, 9, 2.

Press MENU. You will see the dialog box as shown on the right.
Choose 2: STAT and press EXE.



Type the numbers in the first column (List 1).
Press EXE after each number to move down to the next cell.
List 1 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from List 1 to List 26 to enter the list.



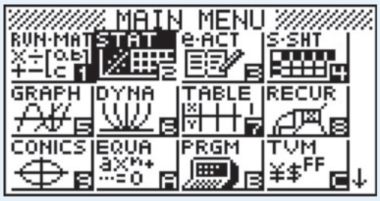
5.2 Entering data from a frequency table

Example 41

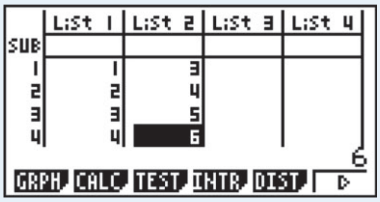
Enter the data in the table:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Press MENU. You will see the dialog box as shown on the right.
Choose the 2: STAT and press EXE.



Type the numbers in the first column (List 1) and the frequencies in the second column (List 2).
Press EXE after each number to move down to the next cell. Press ▶ to move to the next column.
List 1 and List 2 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from List 1 to List 26 to enter the lists.



Drawing charts

Charts can be drawn from a list or from a frequency table.

5.3 Drawing a frequency histogram from a list

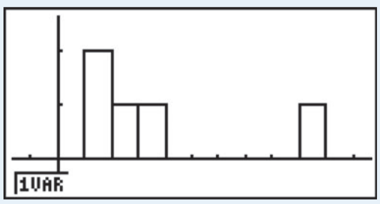
Example 42

Draw a frequency histogram for this data:
1, 1, 3, 9, 2.

Enter the data in List 1 (see Example 40).
Press F1 GRPH and F6 SET.
By default the graph type is Scatter Diagram (Scat). Change this to Histogram (Hist) and leave XList as List 1 and Frequency as 1.

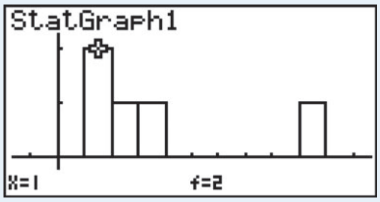
Press EXIT and press F1 to select GPH1.
Change the start value for the histogram to 0 and the width of the bars to 1.
Press EXE.

A histogram, scaled for the data in the list is displayed.



▶ Continued on next page

Press SHIFT F1 (Trace).
Use the ▸ key to move to each of the bars and display their value and frequency.
You should now see a frequency histogram for the data in the list 1, 1, 3, 9, 2.



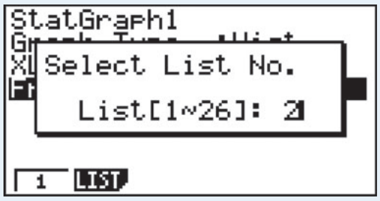
5.4 Drawing a frequency histogram from a frequency table

Example 43

Draw a frequency histogram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in List 1 and List 2 (see Example 41).
Press F1 GRPH and F6 SET.

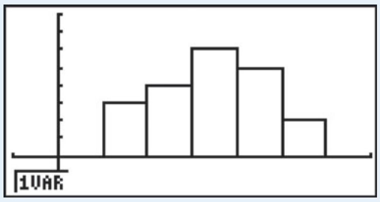


By default the graph type is Scatter Diagram (Scat). Change this to Histogram (Hist). Leave XList as List 1 and set the Frequency as List 2.

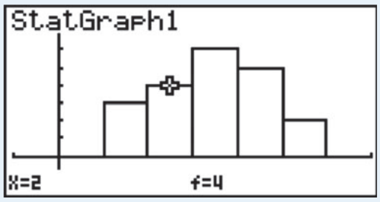
You should see this display.

Press EXIT and press F1 to select GPH1.
Change the start value for the histogram to 0 and the width of the bars to 1.
Press EXE.

A histogram, scaled for the data in the list is displayed.



Press SHIFT F1 (Trace).
Use the ▸ key to move to each of the bars and display their value and frequency.

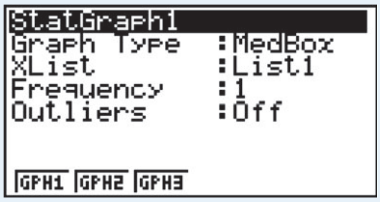


5.5 Drawing a box and whisker diagram from a list

Example 44

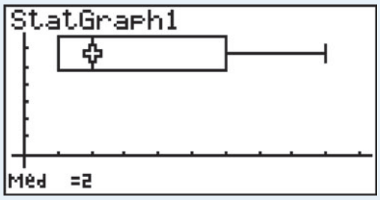
Draw a box and whisker diagram for this data:
1, 1, 3, 9, 2.

Enter the data in List 1 (see Example 40).
Press F1 GRPH and F6 SET.
By default the graph type is Scatter Diagram (Scat). Change this to MedBox (Box) and leave XList as List 1, the Frequency as 1 and Outliers off.



Press EXIT and press F1 to select GPH1.

Press SHIFT F1 (Trace).
Use the ▶ key to move the cursor over the plot to see the quartiles, Q1 and Q3, the median and the maximum and minimum values.



5.6 Drawing a box and whisker diagram from a frequency table

Example 45

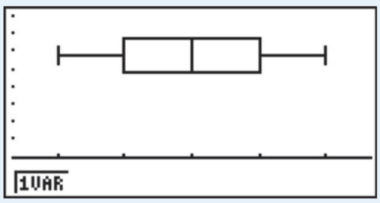
Draw a box and whisker diagram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in List 1 and List 2 (see Example 41).
Press F1 GRPH and F6 SET.
By default the graph type is Scatter Diagram (Scat). Change this to MedBox (Box). Leave XList as List 1, set the Frequency as List 2 and leave Outliers off.

You should see this display.

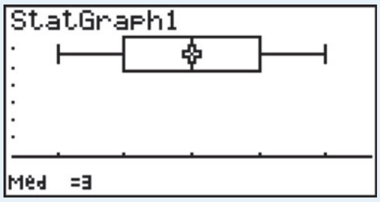
Press EXIT and press F1 to select GPH1.



Continued on next page

Press SHIFT F1 (Trace).

Use the ▶ key to move the cursor over the plot to see the quartiles, Q1 and Q3, the median and the maximum and minimum values.



Calculating statistics

You can calculate statistics such as mean, median, etc. from a list, or from a frequency table.

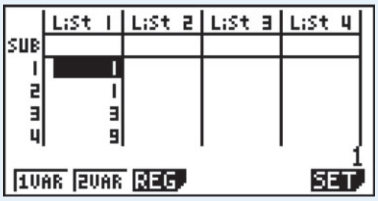
5.7 Calculating statistics from a list

Example 46

Calculate the summary statistics for this data: 1, 1, 3, 9, 2

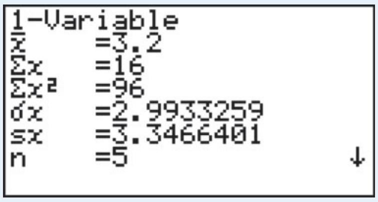
Enter the data in List 1 (see Example 40).
Press F2 CALC and F6 SET.
Use the default values for 1 variable statistics which are XList as List 1 and Freq as 1.

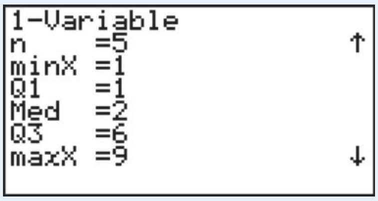
Press EXIT and press F1 to select 1 VAR.

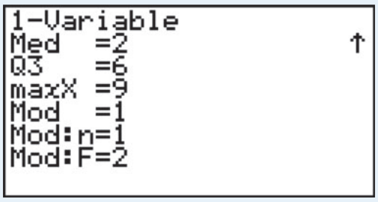


The information shown will not fit onto a single screen.
You can scroll up and down to see it all.
The statistics calculated for the data are:

mean	$\bar{x}$
sum	$\sum x$
sum of squares	$\sum x^2$
population standard deviation	σx
sample standard deviation	Sx
number	n
minimum value	minX
lower quartile	Q_1
median	Med
upper quartile	Q_3
maximum value	maxX
mode	Mod
number of data mode items	Mod:n
data mode frequency	Mod:F







5.8 Calculating statistics from a frequency table

Example 47

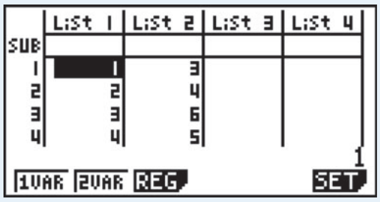
Calculate the summary statistics for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in List 1 and List 2 (see Example 41).
Press F2 CALC and F6 SET.
Leave the 1 variable statistics default value for XList as List 1 and change Freq to List 2.

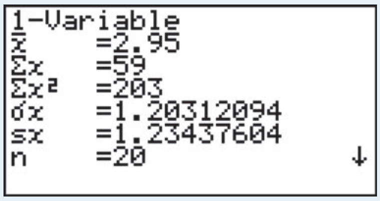
You should see this display.

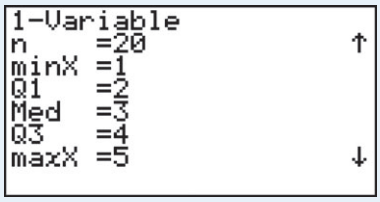
Press EXIT and press F1 to select 1 VAR.

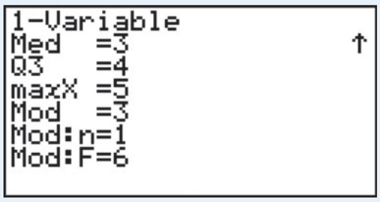


The information shown will not fit onto a single screen.
You can scroll up and down to see it all.
The statistics calculated for the data are:

mean	$\bar{x}$
sum	$\sum x$
sum of squares	$\sum x^2$
population standard deviation	σx
sample standard deviation	Sx
number	n
minimum value	minX
lower quartile	Q_1
median	Med
upper quartile	Q_3
maximum value	maxX
mode	Mod
number of data mode items	Mod:n
data mode frequency	Mod:F







5.9 Calculating the interquartile range

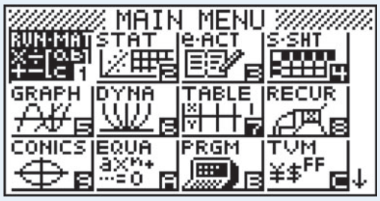
Example 48

Calculate interquartile range for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

The interquartile range is the difference between the upper and lower quartiles ($Q_3 - Q_1$).

Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.



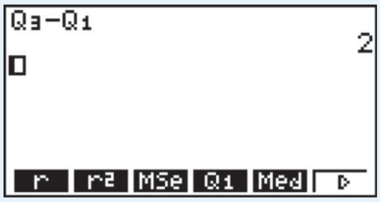
First calculate the summary statistics for this data (see Example 47).

(**Note**: The values of the summary statistics are stored after 1-Variable Statistics have been calculated and remain stored until the next time they are calculated.)

Press VARS | F3 STAT | F3 GRPH | F6 F6 | F1 Q_3 − F6 F6 | F4 Q_1 EXE

The calculator now displays the result:

Interquartile range = $Q_3 - Q_1 = 2$



5.10 Using statistics

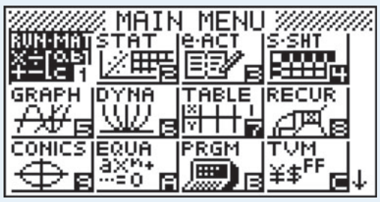
The calculator stores the values you calculate in One-Variable Statistics so that you can access them in other calculations. These values are stored until you do another One-Variable Statistics calculation.

Example 49

Calculate $\bar{x} + \sigma x$ for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.



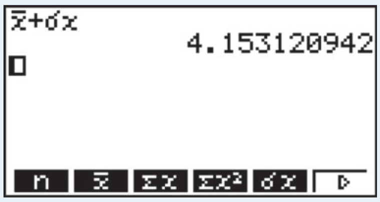
First calculate the summary statistics for this data (see Example 47).

(**Note:** The values of the summary statistics are stored after 1-Variable Statistics have been calculated and remain stored until the next time they are calculated.)

Press VARS | F1 X | F2 $\bar{x}$ + F5 σx EXE

The calculator now displays the result:

$\bar{x} + \sigma x = 4.15$ (to 3 sf)



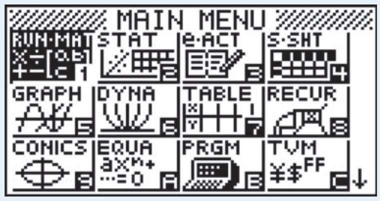
Calculating binomial probabilities

5.11 The use of nCr

Example 50

Find the value of $\binom{8}{3}$ (or ${}_8C_3$).

Press MENU. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press EXE.



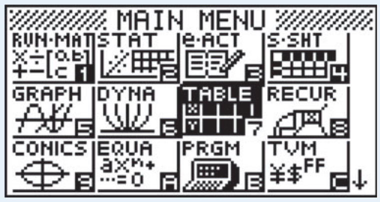
Press OPTN F6 ▸ | F3 PROB to enter probability menu.

Press 8 F3 nCr 3 EXE.

Example 51

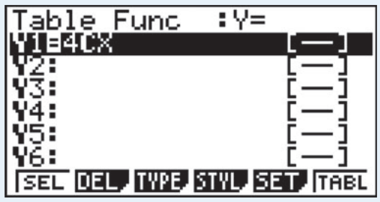
List the values of $\binom{4}{r}$ for $r = 0, 1, 2, 3, 4$.

Press MENU. You will see the dialog box as shown on the right.
Choose 7: TABLE and press EXE.



Press OPTN F4 PROB to enter probability menu.

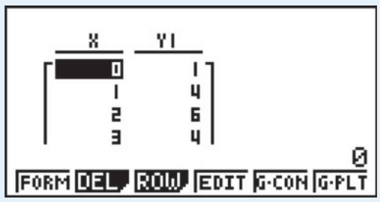
Press 4 F3 nCr X,θ,T EXE.



Press F6 TABL.
The table shows that

$$\binom{4}{0}=1, \binom{4}{0}=1, \binom{4}{1}=4, \binom{4}{2}=6, \binom{4}{3}=4 \text{ and } \binom{4}{4}=1$$

You may need to reset the start value and step values for the table using F5 SET.



5.12 Calculating binomial probabilities

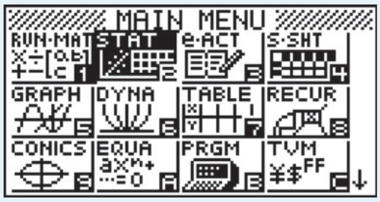
Example 52

X is a discrete random variable and $X \sim \text{Bin}(9, 0.75)$.
Calculate $P(X = 5)$

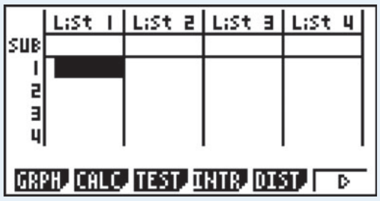
$$P(X=5) = \binom{9}{5} 0.75^5 \, 0.25^4$$

The calculator can find this value directly.

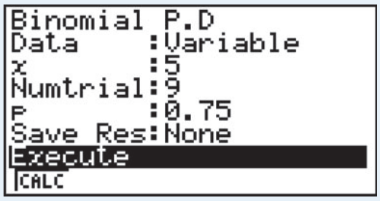
Press MENU. You will see the dialog box as shown on the right.
Choose 2: STAT and press EXE.



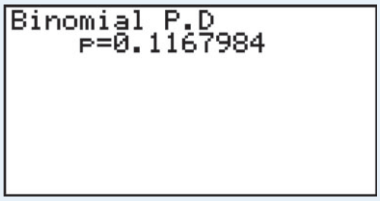
Press F5 DIST | F5 BINM | F1 Bpd.



Enter the x-value 5, the number of trials, 9 and the probability 0.75.
Press EXE.



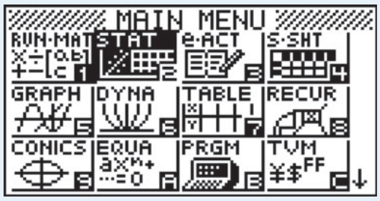
The calculator shows that $P(X = 5) = 0.117$ (to 3 sf).



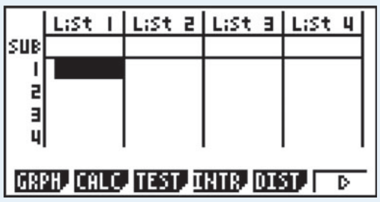
Example 53

X is a discrete random variable and $X \sim \text{Bin}(7, 0.3)$.
Calculate the probabilities that X takes the values $\{0, 1, 2, 3, 4, 5, 6, 7\}$.

Press MENU. You will see the dialog box as shown on the right.
Choose 2: STAT and press EXE.

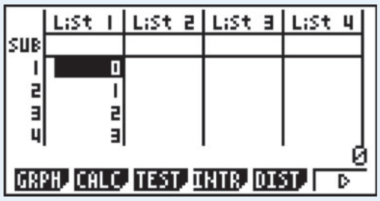


Press F5 DIST | F5 BINM | F1 Bpd.

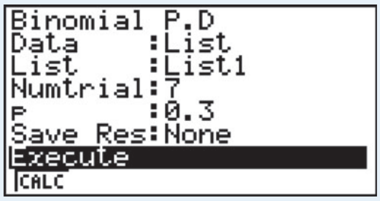


Continued on next page

Enter the values 0,1,2,3,4,5,6,7 in List1. Press EXE after each number.
Press F5 DIST | F5 BINM | F1 Bpd.

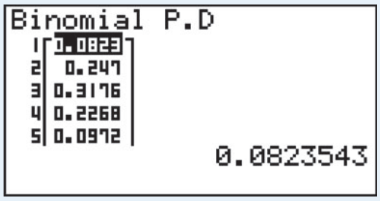


Press F1 List.
Choose List1.
Enter the number of trials, 7 and the probability 0.3.
Press EXE.



The calculator displays each of the probabilities.
To see the remaining values scroll the screen down.

Note: the figures on the left refer to the positions of the values in the list, not the values themselves.



Example 54

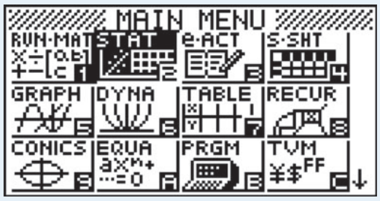
X is a discrete random variable and $X \sim \text{Bin}(20, 0.45)$.
Calculate

a the probability that X is less than or equal to 10.

b the probability that X lies between 5 and 15 inclusive.

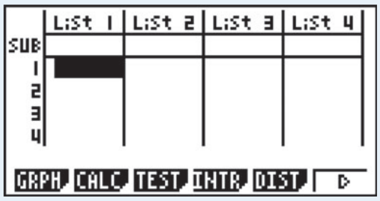
c the probability that X is greater than 11.

Press MENU. You will see the dialog box as shown on the right.
Choose 2: STAT and press EXE.

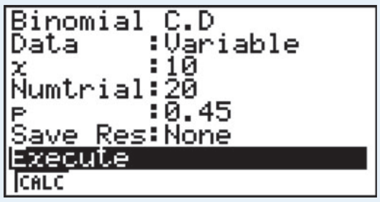


Press F5 DIST | F5 BINM | F2 Bcd.

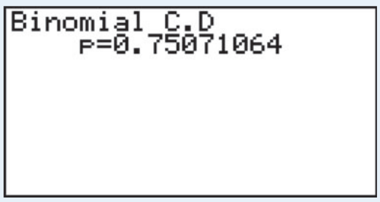
You are given the lower bound probability so you have to calculate other probabilities using this.



a Enter the x-value 5, the number of trials, 9 and the probability 0.75.
Press EXE.

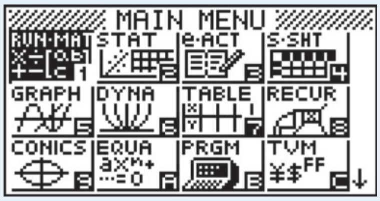


$P(X \le 10) = 0.751$ (to 3 sf)



Continued on next page

b When using the lower bound probability to calculate other probabilities, it is easier to do this without the wizard.
Press MENU. Choose 1: RUN·MAT and press EXE.



$P(5 \le X \le 15) = P(X \le 15) - P(X \le 4)$
Press OPTN F5 STAT | F3 DIST | F5 BINM to enter the binomial menu.
Enter binominalCD(15, 20, 0.45) – binominalCD(4, 20, 0.45)
$P(5 \le X \le 15) = 0.980$ (to 3 sf)

c $P(X > 11) = 1 - P(X \le 11)$
Enter 1 – binominalCD (11, 20, 0.45)
$P(X > 11) = 0.131$ (to 3 sf)

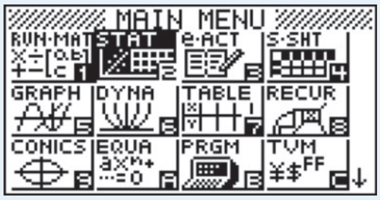
Calculating Poisson probabilities

5.13 Calculating Poisson probabilities

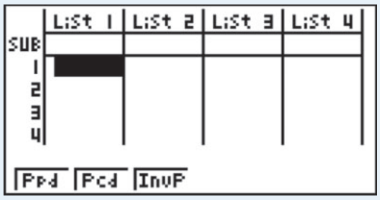
Example 55

X is a discrete random variable and $X \sim Po(0.5)$
Calculate **i** $P(X = 2)$
ii $P(X \le 2)$
iii $P(X > 2)$

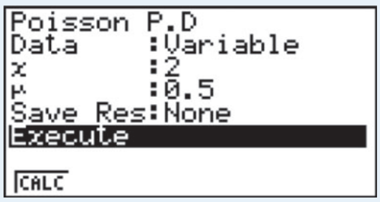
i $P(X = 2) = \dfrac{e^{-0.5} \times (0.5)^2}{2!}$
The calculator can find this value directly.
Press MENU and choose 2: STAT and press EXE.



Press F5 DIST | F6 ▶ | F1 POISN | F1 Ppd to use the Poisson probability density function.

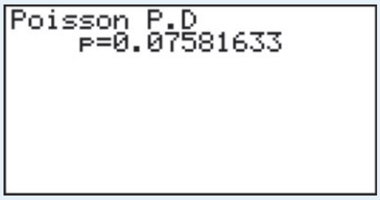


Enter the X value and the parameter.
Leave Data as Variable and Save Res as None.
Select Execute and press EXE.



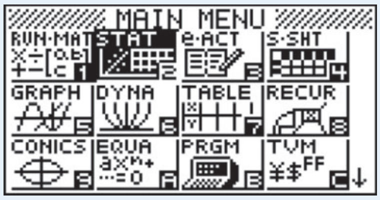
▶ Continued on next page

The calculator shows that
P(X = 2) = 0.0758 (to 3 sf)

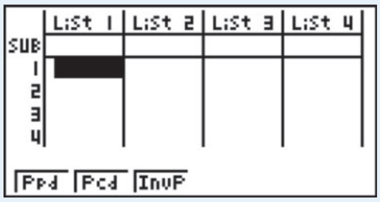


ii $P(X \le 2) = \dfrac{e^{-0.5} \times (0.5)^0}{0!} + \dfrac{e^{-0.5} \times (0.5)^1}{1!} + \dfrac{e^{-0.5} \times (0.5)^2}{2!}$

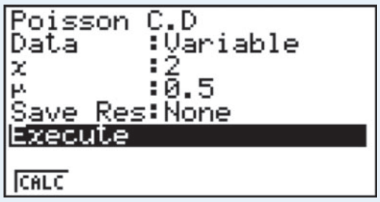
The calculator can find this value directly.
Press MENU, and choose 2: STAT and press EXE.



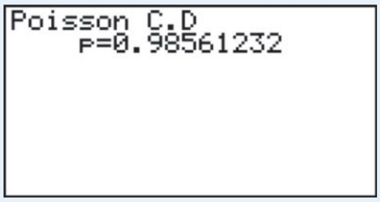
Press F5 DIST | F6 ▸ | F1 POISN | F2 Pcd to use the Poisson cumulative distribution function.



Enter the X value and the parameter.
Leave Data as Variable and Save Res as None.
Select Execute and press EXE.



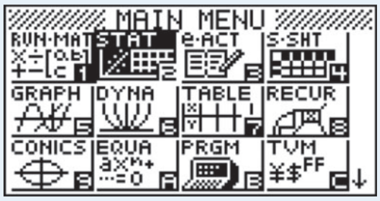
The calculator shows that
P($X \le$ 2) = 0.986 (to 3 sf)



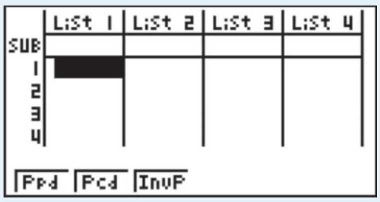
iii $P(X > 2) = \dfrac{e^{-0.5} \times (0.5)^3}{3!} + \dfrac{e^{-0.5} \times (0.5)^4}{4!} + \dfrac{e^{-0.5} \times (0.5)^5}{5!}$

$= 1 - P(X \le 2)$

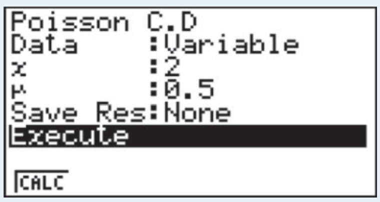
First you need to calculate P($X \le$ 2).
Press MENU, and choose 2: STAT and press EXE.



Press F5 DIST | F6 ▸ | F1 POISN | F2 Pcd to use the Poisson cumulative distribution function.

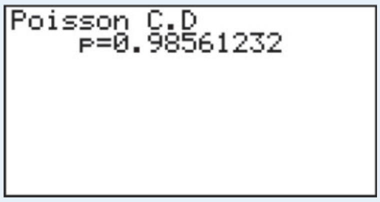


Enter the X value and the parameter.
Leave Data as Variable and Save Res as None.
Select Execute and press EXE.

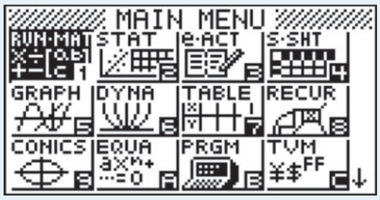


▶ Continued on next page

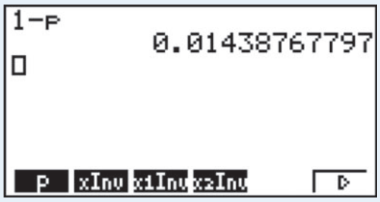
The calculator shows that
$P(X \leq 2) = 0.986$ (to 3 sf)



Once you have calculated $P(X \leq 2)$ you need to find
$1 - P(X \leq 2)$
Press MENU.
Choose 1: RUN · MAT and press EXE.



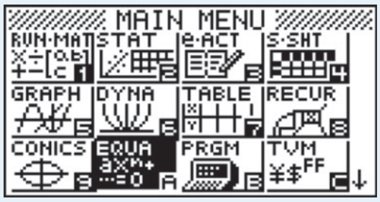
Type 1 − and press VARS | F3 STAT | F6 RESLT | F3 DIST | F1 p
Press EXE
The calculator shows that
P (X > 2) = 0.0144 (to 3 sf)



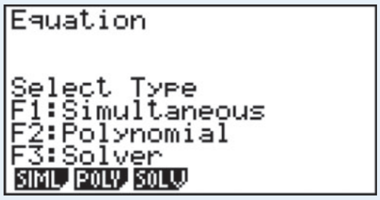
Example 56

If $X \sim \text{Po}(\mu)$ find the value of μ, correct to 3 decimal places, given that $P(X = 2) = 0.035$.
There is no inverse Poisson function on the Casio fx-9860GII, so instead you should use the numerical solver function to find a value of μ when you are given a probability.

Press MENU. You will see the dialog box as shown on the right.
Choose A: EQUA and press EXE.

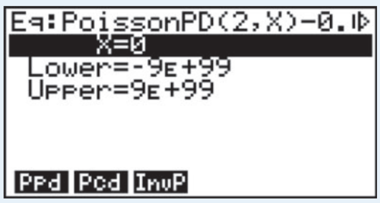


Press F3 to choose the solver.



The solver will numerically solve equations of the type $f(x) = 0$.
Type the equation
PoissonPD(2, x) −0.035
To enter PoissonPD, press OPTN | F6 ▸ | F3 STAT | F1 DIST | F6 ▸ |
F1 POISN | F1 Ppd

Enter an initial guess for x, 0 is close enough.
Press EXIT until you return to the menu that shows F6 SOLV.
Press F6



▶ Continued on next page

The required value of μ is 0.309 (to 3 sf).

Calculating normal probabilities

5.14 Calculating normal probabilities from *X*-values

Example 57

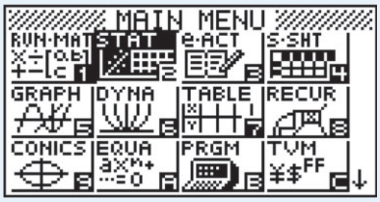
A random variable X is normally distributed with a mean of 195 and a standard deviation of 20 or $x \sim N(195, 20)^2$. Calculate

a the probability that X is less than 190.

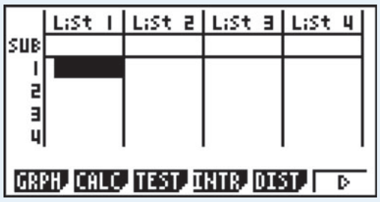
b the probability that X is greater than 194.

c the probability that X lies between 187 and 196.

Press MENU and choose 2: STAT and press EXE.

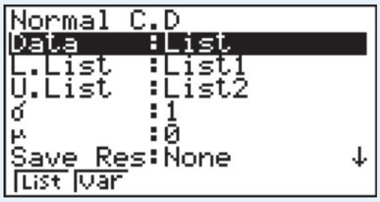


Press F5 DIST F1 NORM F2 Ncd to use the Normal Cumulative Distribution function.



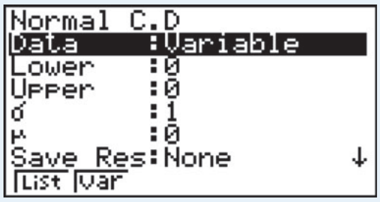
Press F2 Var.

The value 1E99 is the largest value that can be entered in the GDC and is used in the place of ∞. It stands for 1×10^{99} (-1E99 is the smallest value and is used in the place of $-\infty$). To enter the E, you need to press EXP.



This dialogue box is used to calculate normal probabilities.

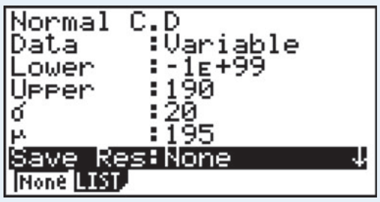
You should enter the values, Lower Bound, Upper Bound, σ and μ, in order.



a $P(x < 190)$

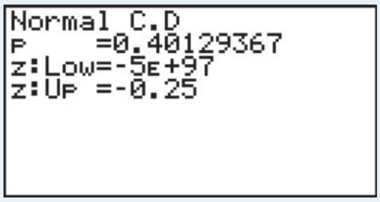
Enter Lower Bound as -1E99, Upper Bound as 190, σ as 20 and μ as 195.

Press EXE.

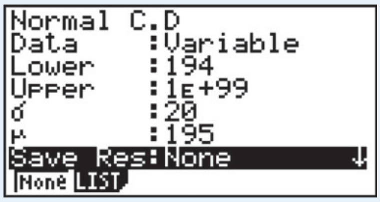


Continued on next page

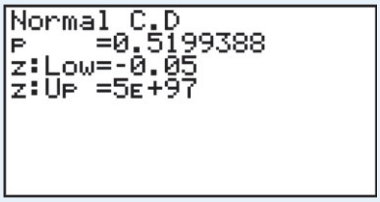
P(X < 190) = 0.401 (to 3 sf)



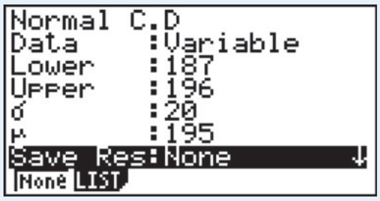
b $P(x > 194)$
Press EXIT to return to the entry screen.
Enter Lower Bound as 194, Upper Bound as 1E99, σ as 20 and μ as 195.



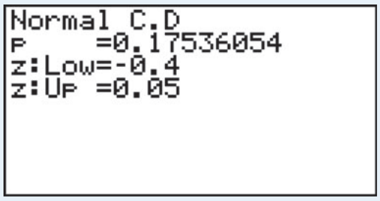
P(X > 194) = 0.520 (to 3 sf)



c $P(187 < x < 196)$
Press EXIT to return to the entry screen.
Enter Lower Bound as 187, Upper Bound as 196, μ as 195 and σ as 20.

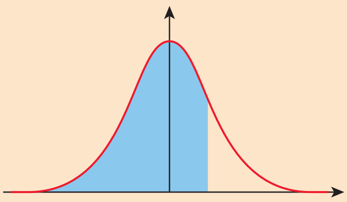


P(187 < X < 196) = 0.175 (to 3 sf)

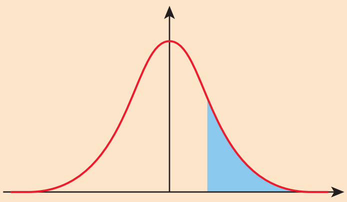


5.15 Calculating *X*-values from normal probabilities

In some problems you are given probabilities and have to calculate the associated values of X. To do this, use the Inverse Normal function.



When using the Inverse Normal function, make sure you find the probability on the correct side of the normal curve. If the area is the lower tail, that is it is in the form $P(X < x)$, use 'Left' tail.



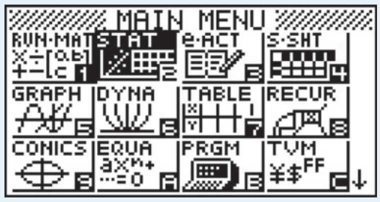
If you are given the upper tail $P(X > x)$, you should use 'Right' tail.

Example 58

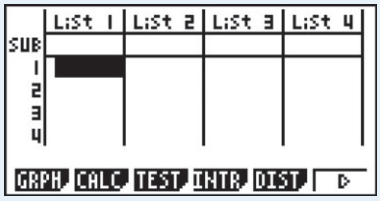
A random variable X is normally distributed with a mean of 75 and a standard deviation of 12 or $X \sim \mathrm{N}(75, 12^2)$. If $P(X < x) = 0.4$, find the value of x.

You are given a lower-tail probability so you should choose 'Left'.

Press MENU and choose 2: STAT and press EXE.

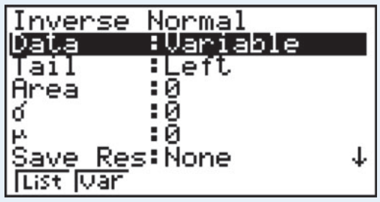


Press F5 DIST F1 NORM F3 InvN to use the Inverse Normal function.

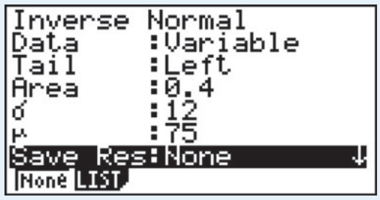


Press F2 Var.

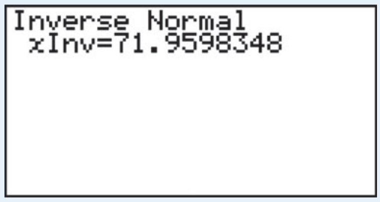
You should enter the values: Area (probability), σ and μ, in order.



Enter Tail as Left (F1), Area as 0.4, σ as 12 and μ as 75.
Press EXE.



So if $P(X < x) = 0.4$ then $x = 72.0$ (to 3 sf).

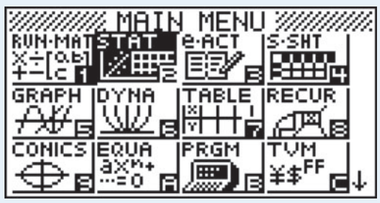


Example 59

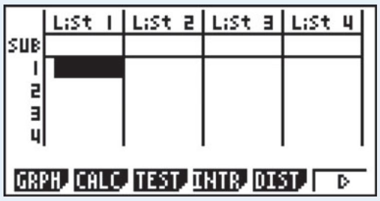
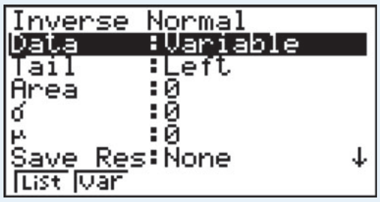
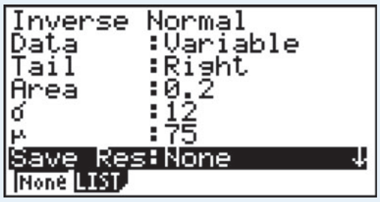
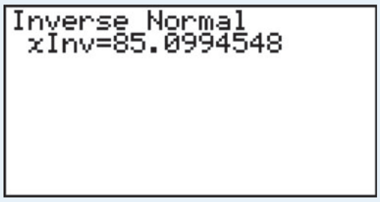
A random variable X is normally distributed with a mean of 75 and a standard deviation of 12 or $X \sim \mathrm{N}(75, 12^2)$.
If $P(X > x) = 0.2$, find the value of x.

You are given an upper-tail probability so you should choose 'Right'.

Press MENU and choose 2: STAT and press EXE.



Continued on next page

Press F5 DIST F1 NORM F3 InvN to use the Inverse Normal function.	
Press F2 Var. You should enter the values: Area (probability), σ and μ, in order.	
Enter Tail as Right (F2), Area as 0.2, σ as 12 and μ as 75. Press EXE.	
So if $P(X > x) = 0.2$ then $x = 85.1$ (to 3 sf).	

This sketch of a normal distribution curve shows the value of x and the probabilities for Example 59.