

Practice paper 1

Time allowed: 2 hours

- Answer all the questions
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

1 Given that $4\ln 2 - 3\ln 4 + \ln k = 0$, find the value of k .

[4 marks]

This image shows a full page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice. There are no margins, text, or other markings on the page.

- 2 a** Show that $p(x) = 2x^3 - 3x^2 + 8x + 5$ is divisible by $2x + 1$.
b Hence find all the zeros of $p(x)$.

[4 marks]

[4 marks]

[illegible]

- 3** The sum of the first two terms of a geometric sequence is $\frac{8}{9}$ and the sum of the first three terms is $\frac{26}{27}$.

[8 marks]

[illegible]

- 4** Consider the events A and B such that $P(A)=0.3$ and $P(B)=0.2$.

Given that $P(A \cup B) = 3P(A \cap B)$, find $P(A|B)$ and $P(A \cap B')$

[6 marks]

[illegible]

5 Show that for any complex number z ,

a $z + z^* = 2\text{Re}(z)$

b $z - z^* = 2i \operatorname{Im}(z)$

c $\operatorname{Re}(z) \leq |z|$

[6 marks]

This image shows a full page of a document template. It consists of approximately 30 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings present.

- 6** Find a vector equation of the line of intersection of the planes with equations $x + 2y - z = 5$ and $-3x - y + z = 1$.

[6 marks]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 7** A curve is defined by the equation $x^2 + 4y^2 - 2x + 16y + 13 = 0$.

Find the coordinates of the points on the curve where the tangent to the curve is parallel to the x -axis.

[6 marks]

[illegible]

- 8** Use integration by parts to find the rational values of a and b such that

$$\int_1^9 \sqrt{x} \ln x \, dx = a \ln 3 + b.$$

[5 marks]

[illegible]

- [7 marks]

[illegible]

10 Consider the function defined by $f(x) = 12 \sin x - 5 \cos x$.

Find the range of f .

[4 marks]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

$$L_1: \frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{3} \text{ and } L_2: \frac{x-1}{4} = \frac{y-2}{1} = \frac{z-3}{-2}$$

- a** Show that the lines intersect and find their point of intersection. [5 marks]
- b** Hence find the equation of the plane that contains both lines. [4 marks]
- c** Show that the point $A(1, -1, 0)$ does not lie on the plane π . [2 marks]
- d** Write down the equation of the line L_3 perpendicular to the plane π that contains the point A . [1 mark]
- e** Hence find the distance from A to the plane π . [7 marks]

This image shows a full page of white paper with horizontal dotted lines, resembling notebook paper. The lines are evenly spaced and run across the width of the page. There is no handwriting or other markings on the paper.

Handwriting practice lines consisting of 30 horizontal dotted lines.

12 a Prove by mathematical induction that

$$0^2 + 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}, \text{ for all } n \in \mathbb{N}.$$

[7 marks]

b Hence find an expression for $3^2 + 6^2 + \dots + (3n)^2$.

[4 marks]

c Given that $A_n = 1^2 + 4^2 + \dots + (3n-2)^2$ and $B_n = 2^2 + 5^2 + 8^2 + \dots + (3n-1)^2$, prove that $A_n + B_n = 6n^3 - n$ and $A_n - B_n = -3n^2$

Hence find A_n and B_n in terms of n .

[9 marks]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

13 Let $f: x \rightarrow e^{\cos x}$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$

- a** State with a reason whether or not the function f is even.
- b** Find $f'(x)$.
- c** Given that the graph of f has a maximum point, find its coordinates.
- d** Show there is a point of inflexion on the graph of f , for $0 < x < \frac{\pi}{2}$ and find its coordinates.
- e** Sketch the graph of f .
- f** A rectangle is drawn so that its lower vertices are on the x -axis and its upper vertices are on the curve $y = e^{\cos x}$ where $-\frac{\pi}{2} < x < \frac{\pi}{2}$
 - i** Write down an expression for the area of the rectangle.
 - ii** Show that there is a positive value $x = a$ for which the area of the rectangle reaches a maximum.

[2 marks]

[2 marks]

[5 marks]

[6 marks]

[1 mark]

- i Write down an expression for the area of the rectangle.
- ii Show that there is a positive value $x = a$ for which the area of the rectangle reaches a maximum.

[1 mark]

Hence show that its value is given by $2ae^{\frac{\sqrt{a^2-1}}{a}}$

[4 marks]

Handwriting practice area with 30 horizontal dotted lines.